Computational Fluid Dynamics (CFD)

Introductory Course

Session 01 – Basic Concepts of CFD

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March 2023

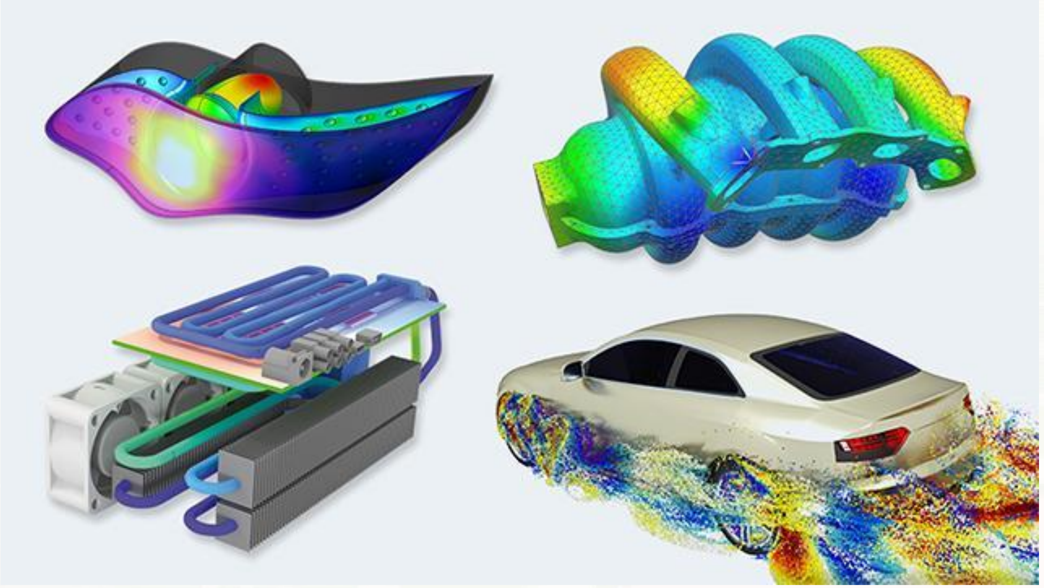
Syllabus :

- Session 01 – Basic Concepts of CFD
- Session 02 – Review of Vector Calculus
- Session 03 – Introduction to Numerical Methods
- Session 04 – Mathematical Description of Physical Phenomena – Part 01
- Session 05 – Mathematical Description of Physical Phenomena – Part 02

1. What is CFD ?

- **Definition**. A branch of Fluid Mechanics that uses numerical methods and algorithms to solve and analyze problems that involves fluid flows

- Governing equations of fluid mechanics include :
  - Continuity eq.
  - Momentum eq.
  - Energy eq.



2. Introduction to CFD

- Typical steps of a the CFD Process :

Problem Definition

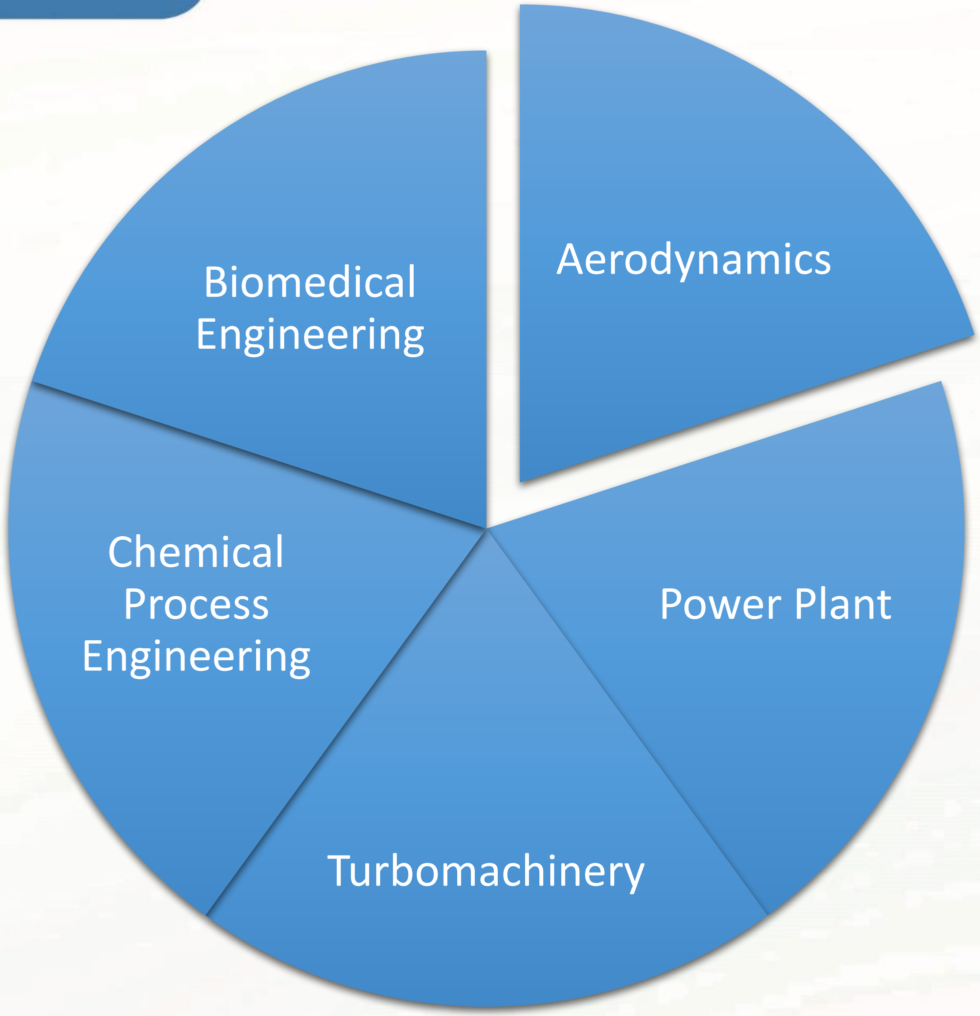
Mesh Generation

Solver Setup

Solution

Post-Processing

3. Applications of CFD



4. Adv. Vs. Disadv. of CFD

Advantages

- Cost-Effective
- Flexibility
- Accuracy
- Visualization
- Optimization

Limitations

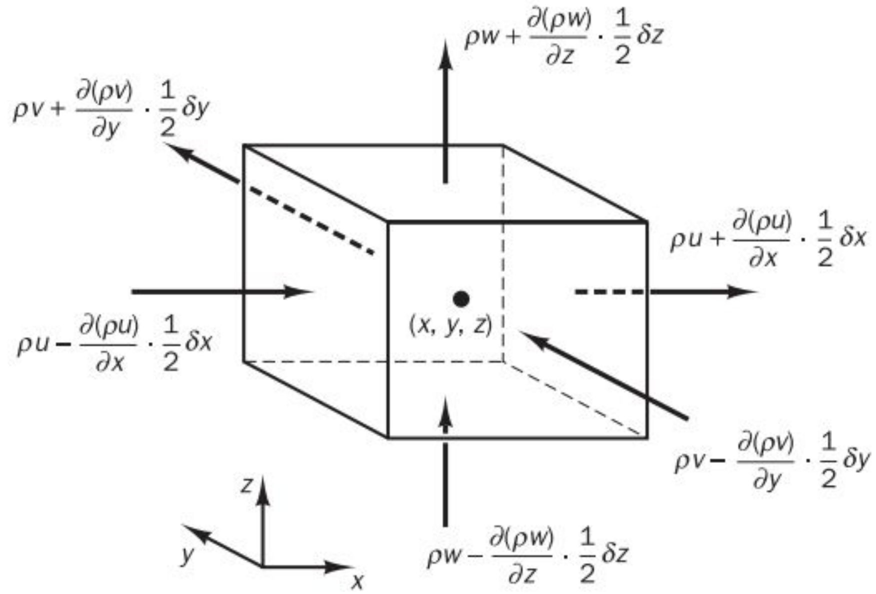
- Boundary Conditions
- Numerical Errors
- Complexity
- Validation
- Assumptions

5. Governing eqs.

- Continuity Eq. (Mass Conservation Law)

  - The General case.
  - Simple forms
    - Incompressible fluid
    - Steady State flow

Rate of increase of mass in fluid element	=	Net rate of flow of mass into fluid element



$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{u}) = 0$$

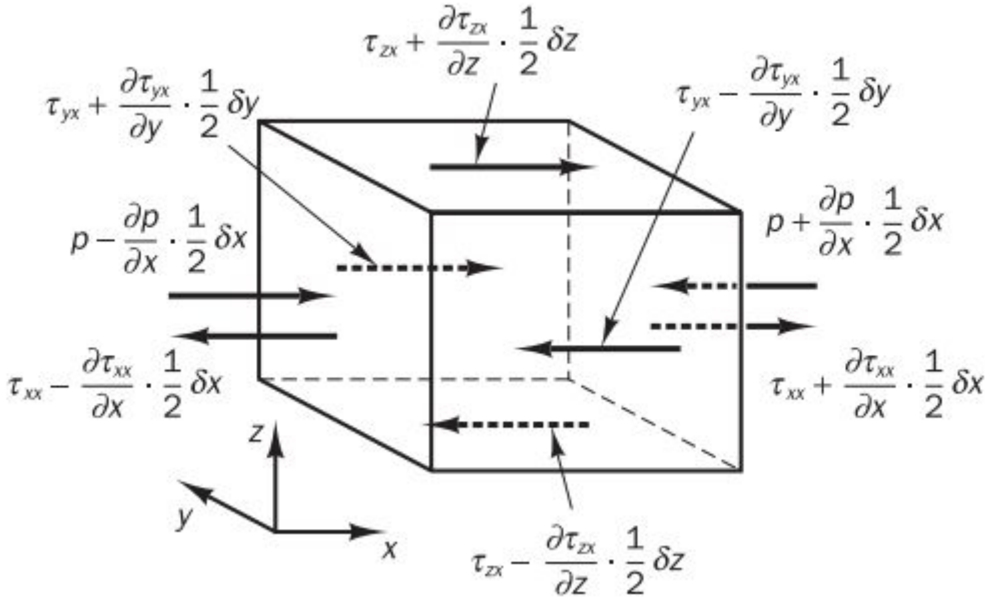
5. Governing eqs.

Rate of increase of momentum of fluid particle	=	Sum of forces on fluid particle

- Momentum Eq. (Linear Momentum Conservation Law)

  - The General case.

- **surface forces**
  - pressure forces
  - viscous forces
  - gravity force
- **body forces**
  - centrifugal force
  - Coriolis force
  - electromagnetic force

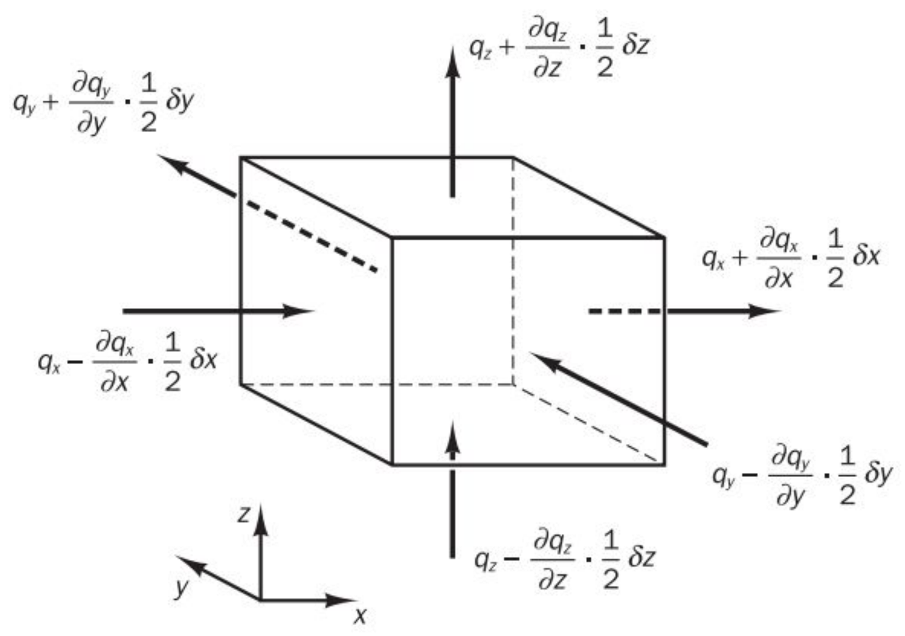


$$\frac{\partial}{\partial t}[\rho \mathbf{v}] + \nabla \cdot \{\rho \mathbf{v}\mathbf{v}\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}_b$$

5. Governing eqs.

Rate of increase of energy of fluid particle	=	Net rate of heat added to fluid particle	+	Net rate of work done on fluid particle

- Energy Eq. (Energy Conservation Law)
  - The General case.

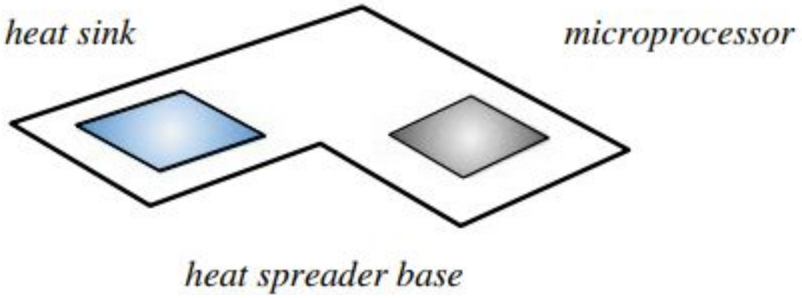


$$\frac{\partial}{\partial t}(\rho c_p T) + \nabla \cdot [\rho c_p \mathbf{v} T] = \nabla \cdot [k \nabla T] + Q^T$$

6. Numerical Methods

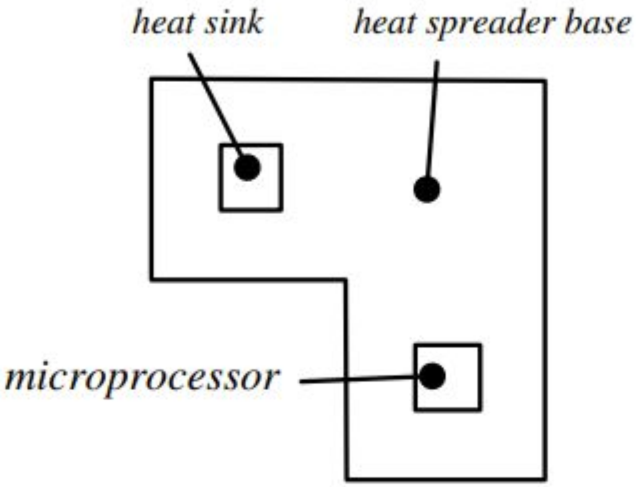
- Finite Difference Methods
  - Discretizes the domain into a grid
- Finite Element Methods
  - Discretizes the domain into small element
- Finite Volume Methods
  - Discretizes the domain into small control volumes
- Spectral Methods
  - Based on approximation of the solution of the PDE using a basis function
- Boundary Element Methods
  - Puts a focus on exterior region of the physical domain
- Galerkin Methods
  - Forefather of FEM
- Lattice-Boltzmann Methods

7. Steps to solving

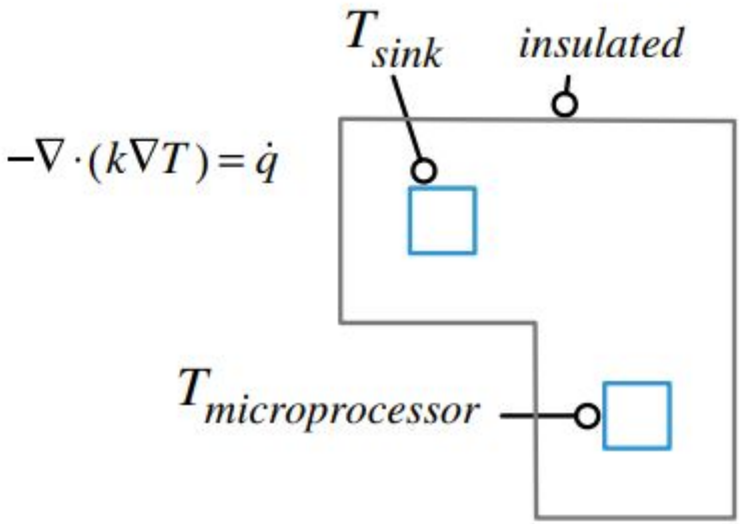


- Step 1. " Geometric and Physical Modeling "

Domain Modeling

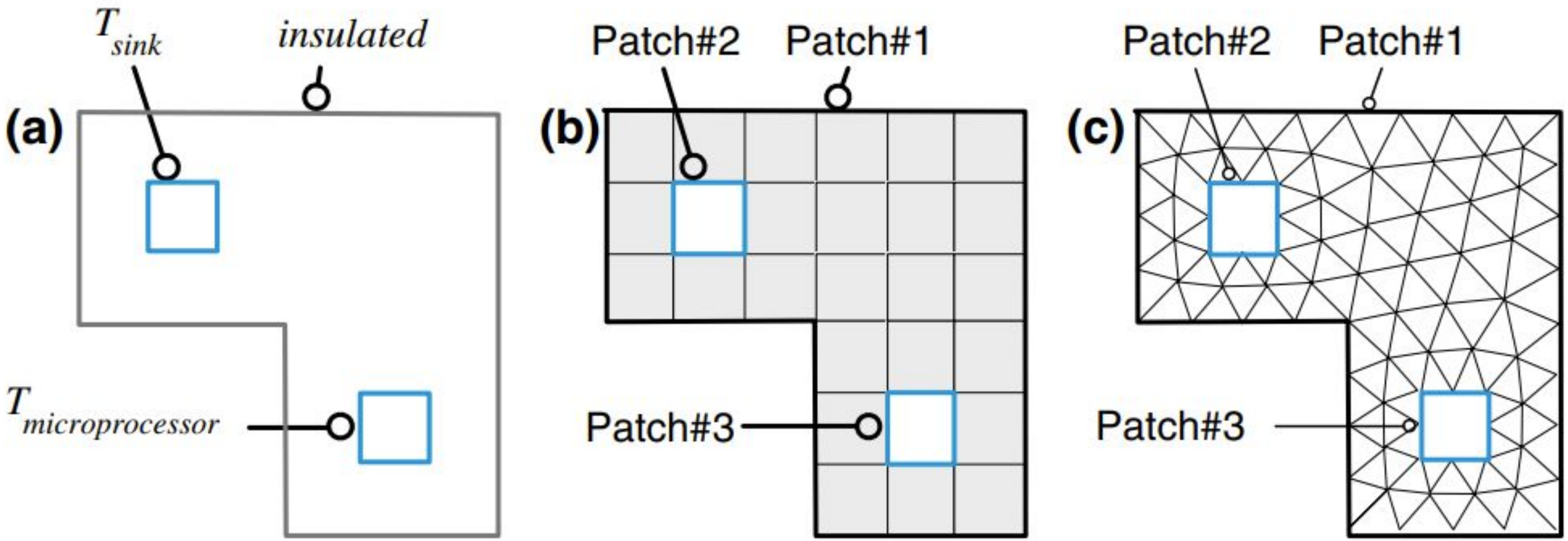


Physical Modeling



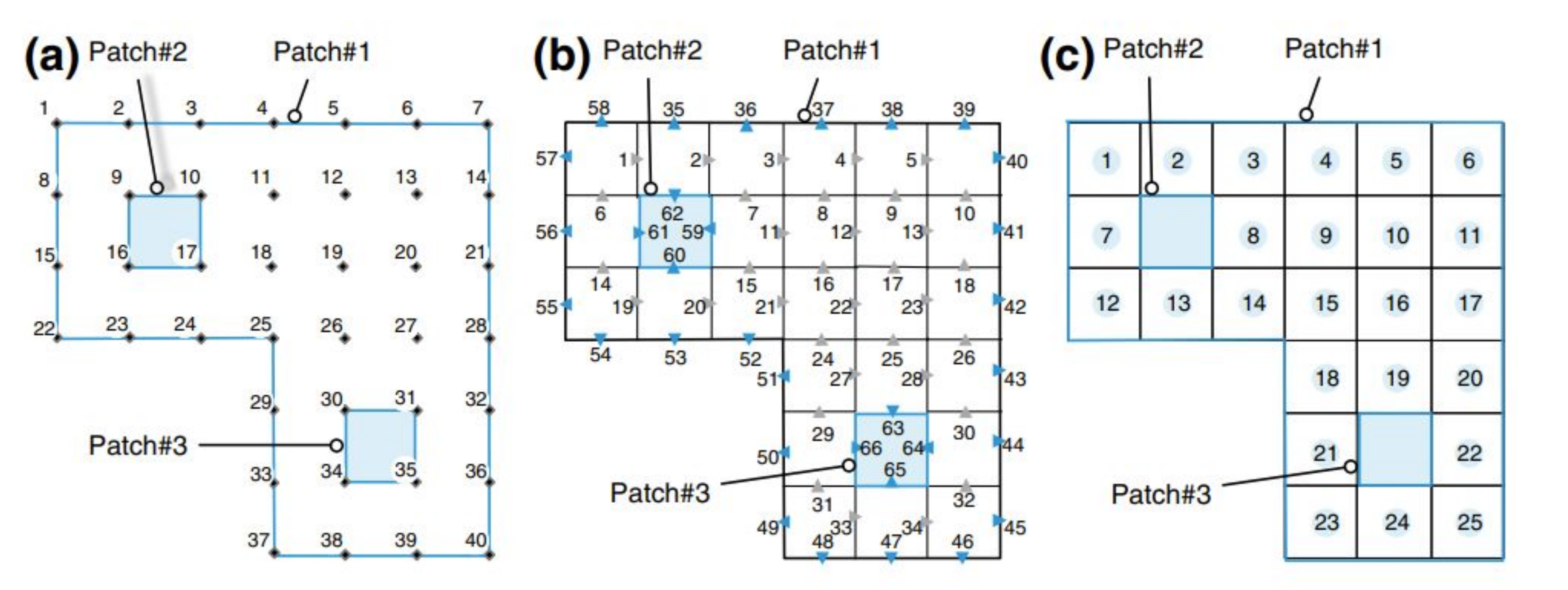
7. Steps to solving

- Step 2. “ Domain Discretization (Mesh Generation) “



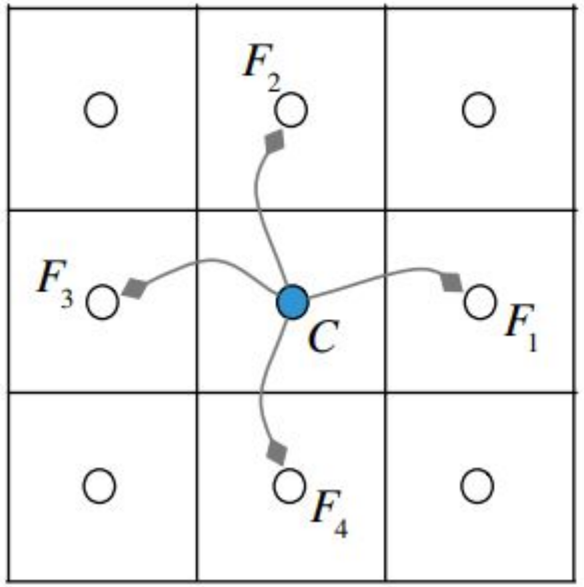
7. Steps to solving

- Step 2. “ Domain Discretization (Mesh Generation) “ – Mesh Topology



7. Steps to solving

- Step 2. “ Domain Discretization (Mesh Generation) “ – Element Connectivity

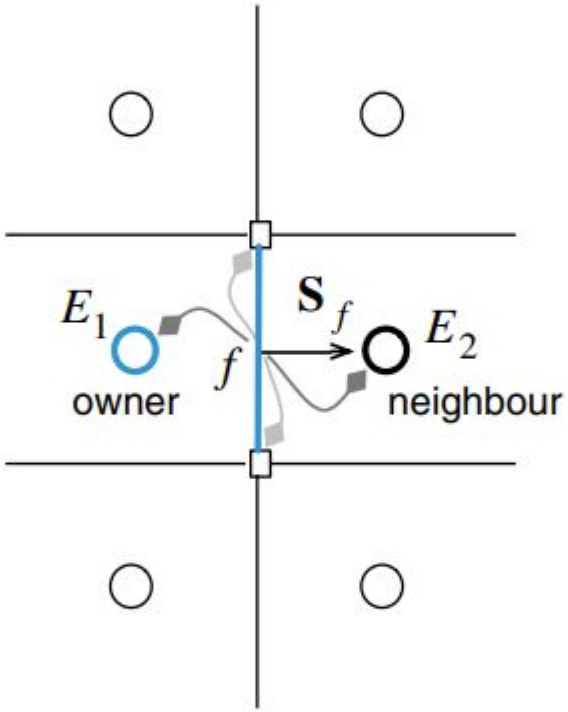


Element 9 Connectivity

Neighbours [10 4 8 15]
Faces [12 8 11 16]
Vertices [19 11 12 18]

7. Steps to solving

- Step 2. " Domain Discretization (Mesh Generation) " – Face Connectivity



Face 12 Connectivity

Element1 9

Element2 10

Vertices [19 12]

7. Steps to solving

- Step 3. “ Equation Discretization (Numerical Approximation) “

$$-\sum_{f\sim nb(C)} (k\nabla T)_f \cdot \mathbf{S}_f = \dot{q}_C V_C$$

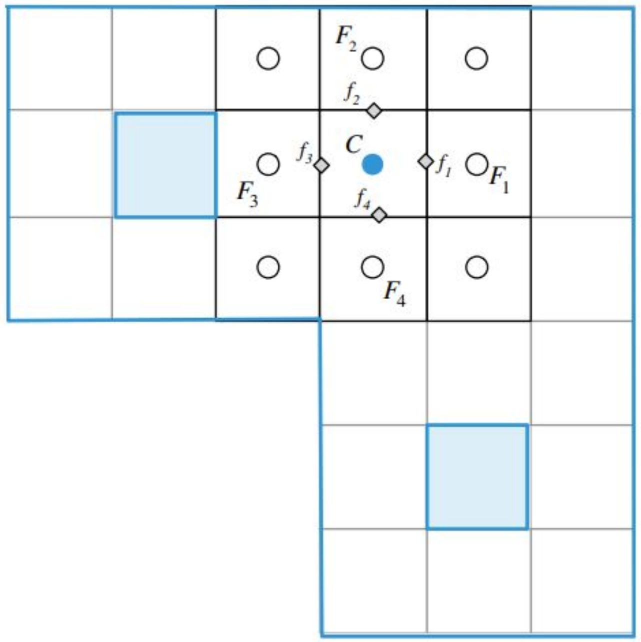
$$-(k\nabla T)_{f_1} \cdot \mathbf{S}_{f_1} - (k\nabla T)_{f_2} \cdot \mathbf{S}_{f_2} - (k\nabla T)_{f_3} \cdot \mathbf{S}_{f_3} - (k\nabla T)_{f_4} \cdot \mathbf{S}_{f_4} = \dot{q}_C V_C$$

$$\mathbf{S}_{f_1} = \Delta y_{f_1} \mathbf{i}$$

$$\delta x_{f_1} = x_{F_1} - x_C$$

$$\nabla T_{f_1} = \left(\frac{\partial T}{\partial x}\right)_{f_1} \mathbf{i} + \left(\frac{\partial T}{\partial y}\right)_{f_1} \mathbf{j}$$

$$\nabla T_{f_1} \cdot \mathbf{S}_{f_1} = \frac{T_{F_1} - T_C}{\delta x_{f_1}} \Delta y_{f_1}$$

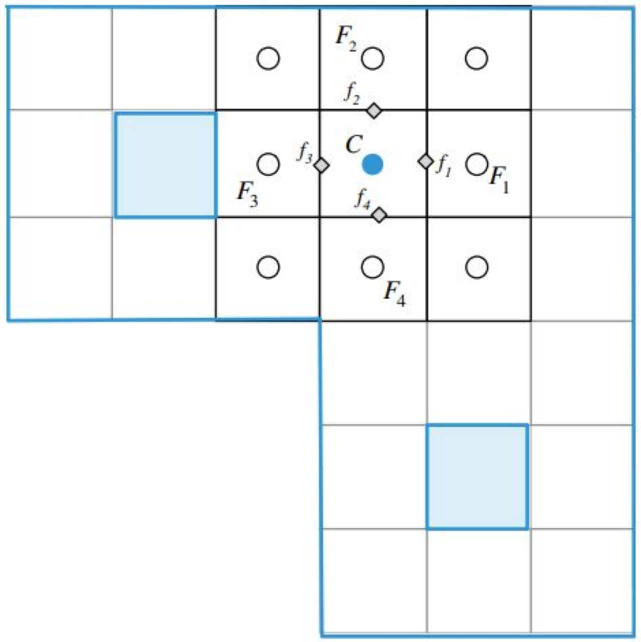


7. Steps to solving

- Step 3. " Equation Discretization (Numerical Approximation) "

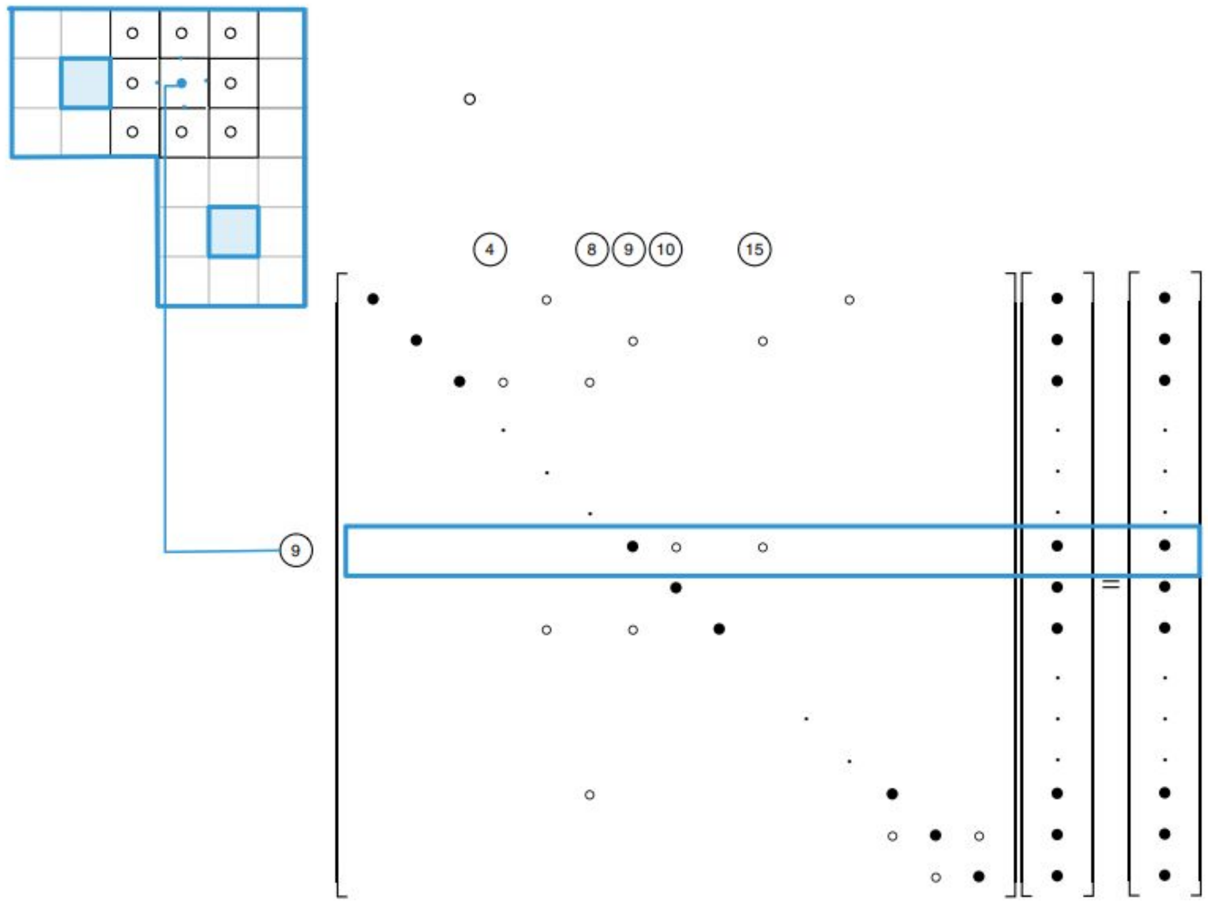
$$-(k\nabla T)_{f_1} \cdot \mathbf{S}_{f_1} = a_{F_1}(T_{F_1} - T_C) \qquad a_{F_1} = -k\frac{\Delta y_{f_1}}{\delta x_{f_1}}$$

$$-\sum_{f\sim nb(C)} (k\nabla T)_f \cdot \mathbf{S}_f = \sum_{F\sim NB(C)} a_F(T_F - T_C)$$
$$= -(a_{F_1} + a_{F_2} + a_{F_3} + a_{F_4})T_C + a_{F_1}T_{F_1} + a_{F_2}T_{F_2} + a_{F_3}T_{F_3} + a_{F_4}T_{F_4}$$
$$= \dot{q}_C V_C$$



7. Steps to solving

- Step 3. “ Equation Discretization (Numerical Approximation) “



7. Steps to solving

- Step 4. “ Solution of the Discretized Equation “

$$\mathbf{A}[T] = \mathbf{b}$$

  - Direct Methods

$$[T] = \mathbf{A}^{-1}\mathbf{b}$$

  - Iterative Methods

$$T_C = \frac{- \sum_{F \sim NB(C)} a_F T_F + b_C}{a_C}$$

8. Importance of Mathematics

- Partial Differential Equations (PDEs)

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \qquad \text{(B.6-1)}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \qquad \text{(B.6-2)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad \text{(B.6-3)}$$

Thanks for your
time and attention