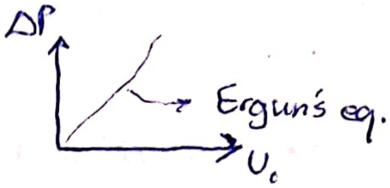
Part II: Engineering Basis of FBs

Chap. 3: Fluidization and Mapping of Regimes

1) Fixed Beds of Particles

1.1 Characterization of Particles

- from ΔP $(\Delta P$ vs. $U_0)$ & Ergun's eq. $\xrightarrow{\text{fitting}}$ d_{eff} ✓



- $d_{sph} \equiv$ [diameter of sphere having the same volume as the particle] $\rightarrow$

$$V_p = \frac{\pi d_{sph}^3}{6}$$

- Sphericity: $\phi_s \equiv \left[\frac{\text{surface of sphere}}{\text{surface of particle}}\right]_{\text{of same volume}}$ (1) $, 0 < \phi_s < 1$

ϕ_s is given in Table 1. p.62

$d_{eff} = \phi_s d_{sph}$ (2)

- specific Surface Area (per particle) $\rightarrow$

$$a' \equiv \left[\frac{\text{surface of a particle}}{\text{volume of a particle}}\right] \quad (3)$$

$$a' = \frac{\pi d_{sph}^2 / \phi_s}{\pi d_{sph}^3 / 6} = \frac{6}{\phi_s d_{sph}} \quad (4)$$

and for the whole bed (a):

ε_m porosity

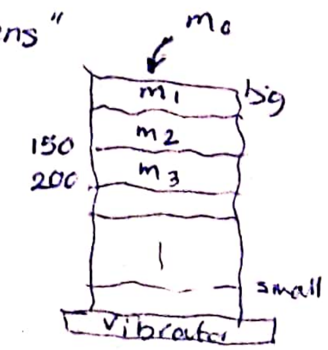
$$a = \frac{6(1-\varepsilon_m)}{\phi_s d_{sph}} \quad (5) \quad [L^2/L^3]$$

- $\bar{d}_p = ?$ "Screen Analysis" $\longrightarrow$ "Tyler Standard Screens"

$\downarrow$

Mean Particle Diameter

Table 2. p. 63



$$\frac{m_3}{m_0} \longrightarrow -150+200$$

$$\Rightarrow \bar{d}_p = \frac{104+74}{2} = 89 \ \mu m$$

1.2 Fixed Beds - One Size of Particles

- ΔP_{fric} for a Fixed Bed $(L_m, \varepsilon_m, d_p, \phi_s)$.

Ergun's eq: $$\frac{\Delta P_{fric}}{L_m} = 150\,\frac{(1-\varepsilon_m)^2}{\varepsilon_m^3}\,\frac{\mu_g U_o}{(\phi_s d_p)^2} + 1.75\,\frac{(1-\varepsilon_m)}{\varepsilon_m^3}\,\frac{\rho_g U_o^2}{\phi_s d_p} \quad (7)$$

$$\Delta P_{measured} = \Delta P_{fric} \pm \rho_g L_m g \quad (8)$$ "+" stands for upflow of gas

1.3 Solids with a PSD

$P \longrightarrow$ cumulative distribution of solids [-]

$p \xrightarrow{d(d_p)}$ volume fraction of particles of size between d_p $[L^{-1}]$ and $d_p + d(d_p)$.

$$P_1 = \int_0^{d_{p_1}} p\, d(d_p) \;,\quad p_1 = \left.\frac{dP}{d(d_p)}\right|_1 \quad (9) \quad \text{(for continuous)}$$

$$P_i = \sum_{1}^{i} p(\Delta d_p)_i = \sum_i x_i \;,\quad p_i = \left(\frac{\Delta P}{\Delta(d_p)}\right)_i \quad \text{(for discrete PSD)} \quad (10)$$

$$x_1 = \frac{m_1}{m_o} \;@\; \bar{d}_{p_1} \;, \ldots$$

$$\bar{a}' = \int_{0 \to d_{p_{min}}}^{d_{p_{max}}} a' p\, d(d_p) \quad (11) \quad = \int_0^{d_{p_{max}}} \left(\frac{6}{\phi_s d_p}\right) p\, d(d_p) \quad (12)$$

$$\bar{a}' = \sum_{all\ i} a_i' (p\,\Delta(d_p))_i \quad (13) \quad = \frac{6}{\phi_s}\sum_{all\ i}\left(\frac{p\Delta(d_p)}{d_p}\right)_i = \frac{6}{\phi_s}\sum_{all\ i}\left(\frac{x_i}{d_{p_i}}\right) = \frac{6}{\phi_s \bar{d}_p} \quad (14)$$

$$(13),(14) \Longrightarrow \bar{d}_p = \frac{6}{\phi_s \bar{a}'} \Longrightarrow \bar{d}_p = \frac{1}{\sum_{all\ i}\left(\frac{x}{d_p}\right)_i} \quad (15) \quad \boxed{\checkmark\checkmark}$$

$\boxed{1.4}$ Experimen of $\phi_{s,eff}$

- For Fixed beds $\longrightarrow$ Ergun's eq. (ϕ_s & $\bar{d}_p$) $\longrightarrow$ $\left\{ \begin{array}{l} \varepsilon_m = \text{fractional voidage} \\ \quad\ \text{void fraction} \\ \text{experimental } \checkmark \end{array} \right.$
- Experimental Procedure ($\phi_{s,eff} = ?$)

1) ε_m ✓ exp.
2) $\Delta P_{fr.}$ @ U_o :

U_o	U_{o1}	U_{o2}	—
$\Delta P_{fr.}$	✓	✓	—

3) $\bar{d}_p$ experimental
4) fitting exp. data using Ergun's eq. $\Rightarrow \phi_s = \cdots = \phi_{s,eff}$

$$\Rightarrow d_{p_{eff}} = \phi_{s,eff} \times \bar{d}_p$$

- ex.1 (p.68)

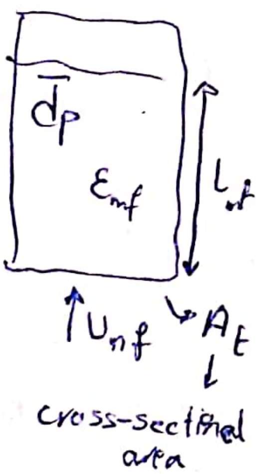
(2) Fluidization without Carryover of Particles

$\boxed{2.1}$ Min fluidization velocity, u_{mf}

@ U_{mf} we have:

$$\left\{ \begin{array}{l} \text{drag force by upward} \\ \text{moving gas} \end{array} \right\} = \left\{ \begin{array}{l} \text{Net weight of} \\ \text{solid particles} \end{array} \right\}$$

for G-S FBs



$$\Longrightarrow (\Delta P_b \cdot A_t) = W_{eff} = (A_t \cdot L_{mf})(1-\varepsilon_{mf})(\rho_p - \rho_g)g \quad (1)$$

$$\Rightarrow \boxed{\frac{\Delta P_b}{L} = (1-\varepsilon_{mf})(\rho_p - \rho_g)g} \quad (2)$$

for $U_o > U_{mf}$:

$$L_m(1-\varepsilon_m) = L_{mf}(1-\varepsilon_{mf}) = L(1-\varepsilon) \quad (3)$$

$\varepsilon_{mf} > \varepsilon_m \longrightarrow$ generally: $\varepsilon_{mf} \simeq \varepsilon_m$

ε_{mf} given in table 3

$- U_{mf} = ?$ $\quad \Delta P_b / L_{mf}$ (Ergun's eq) $= \Delta P_b / L_{mf}$ (2)

$$\Longrightarrow \frac{1.75}{\phi_s \varepsilon_{mf}^3}\left(\frac{d_p u_{mf} \rho_g}{\mu}\right)^2 + \frac{150(1-\varepsilon_{mf})}{\varepsilon_{mf}^3 \phi_s^2}\left(\frac{d_p u_{mf} \rho_g}{\mu}\right) = \frac{\bar{d}_p^3 \rho_g (\rho_s - \rho_g) g}{\mu^2} \quad (5)$$

$$Re_{p,mf} = \frac{\rho_g U_{mf} \bar{d}_p}{\mu} \quad \& \quad Ar = \frac{\bar{d}_p^3 \rho_g (\rho_p - \rho_g) g}{\mu^2} \quad (6)$$

(5), (6)

$$\Longrightarrow \underbrace{\frac{1.75}{\phi_s \varepsilon_{mf}^3} Re_{p,mf}^2}_{\text{inertial}} + \underbrace{\frac{150(1-\varepsilon_{mf})}{\phi_s^2 \varepsilon_{mf}^3} Re_{p,mf}}_{\text{viscous}} = Ar \quad (7) \quad \boxed{\checkmark\checkmark}$$

- for known $\{\bar{d}_p, \phi_s, \rho_p, \rho_g, \mu_g, \varepsilon_{mf}\}$ $\xrightarrow{(7)}$ $Re_{p,mf} \Longrightarrow u_{mf} = \dfrac{\mu Re_{p,mf}}{\rho_g \bar{d}_p}$

Simplification based on $Re_{p,mf}$

I) In the special case of very small particles ($Re_{p,mf} < 20$)

$$(7) \Longrightarrow U_{mf} \cong \frac{\bar{d}_p^2 (\rho_p - \rho_g) g}{150 \mu} \cdot \frac{\varepsilon_{mf}^3 \phi_s^2}{1 - \varepsilon_{mf}} \quad (8)$$

II) for very large particles ($Re_{p,mf} > 1000$)

$$(7) \Longrightarrow U_{mf}^2 \cong \frac{\bar{d}_p (\rho_p - \rho_g) g}{1.75 \rho_g} \varepsilon_{mf}^3 \phi_s \quad (9)$$

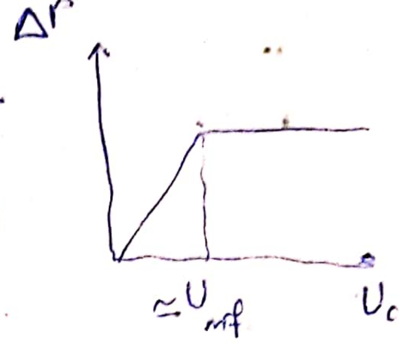
for unknown $\{\varepsilon_{mf} \;\&\; \phi_s\} \longrightarrow u_{mf} = ?$

$$(7) \Longrightarrow \left| \begin{array}{l} k_1 Re_{p,mf}^2 + k_2 Re_{p,mf} = Ar \quad (10) \\ \text{where: } k_1 = \dfrac{1.75}{\varepsilon_{mf}^3 \phi_s} \;\&\; k_2 = \dfrac{150(1-\varepsilon_{mf})}{\phi_s^2 \varepsilon_{mf}^3} \quad (11) \end{array} \right.$$

- Yu & Wen, for ($Re_p = 0.001$ to 4000) : $k_1 \;\&\; k_2 = cte$

k_1 & k_2 given in table 4

- In the simulation & experimental works:



for viscous regime: $\Delta P_{fric.} \propto U_c$ (slope = 1)

$U_c > U_{mf} \implies \Delta P_{fric.} = L(1-\varepsilon)(\rho_p - \rho_g)g = cte$ (12)

Transition from smooth to bubbling fluidization:

U_{mf}, ε_{mf} ⟷ U_{mb}, ε_{mb}

$\begin{cases} L-S & \text{(smooth)} \\ G-S & \text{(smooth + Bubbling)} \end{cases}$

$U_{mb} = ?$ (Emiprical Relation):

$$\frac{U_{mb}}{U_{mf}} = \frac{2300\, \rho_g^{.13}\, \mu^{.52} \exp(.72\, P_{45})}{d_p^{.8} (\rho_p - \rho_g)^{.93}} \quad (13)$$

where $P_{45} = \int_0^{45\,\mu m} p\, d(d_p) = \sum_0^{45\,\mu m} p\, \Delta(d_p)$ fraction of solids smaller than $45\,\mu m$

- In a FB: from a fixed bed: $L_m(1-\varepsilon_m) = L_{mf}(1-\varepsilon_{mf}) = L_{mb}(1-\varepsilon_{mb})$

- ex. 2 (p. 76)

3) Geldart Classification

[4 Groups: C, A, B, D]

- Group C (Cohesive)
  - → $d_p < 50\,\mu m$
  - → very fine powders
  - → Van der Waals forces > Drag force
  - → Normal fluidization ?? ✗

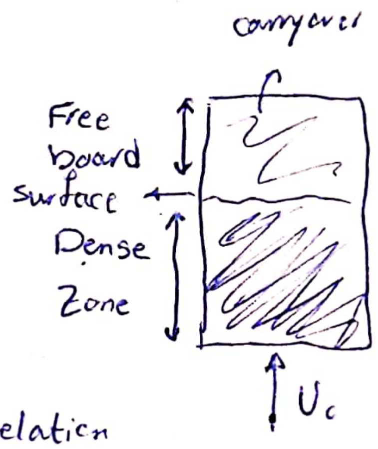
- Group A (Aeratable)
  - → $50\,\mu m < d_p < 100\,\mu m$ & $\rho_p < 1400\, kg/m^3$
  - → smooth fluidization
  - → $\frac{U_{mb}}{U_{mf}} \gg 1$
  - → e.g.: FCC catalyst

- Group B → $50 < \bar{d}_p < 500\ \mu m$ & $1400 < \rho_p < 4000\ kg/m^3$
  → Normal Fluidization → bubbling ✓

- Group D (spoutable)

(4) Fluidization with carry over of particles

4.1 Estimation of Terminal velocity, U_t



- $\sum F_p = 0$, $C_D = \dfrac{24}{Re_p}$ (1) → for stokes regime

$$\rightarrow u_t = \frac{4 d_p \Delta\rho g}{3 \rho_g C_D} \quad (2)$$

- Levenspiel & Haider → proposed an empirical correlation for C_D for all flow regimes (including ϕ_s)

$$C_D = \frac{24}{Re}\left[1 + (8.1716 \exp(-4.0655\phi_s))\, Re_p^{.0964 + .5565\phi_s}\right] + \frac{73.69 \exp(-5.0655\phi_s)\, Re_p}{Re_p + 5.378 \exp(6.2122\phi_s)} \quad (3)$$

- Non-Dimensional forms:

$$d_p^* = d_p\left[\frac{\rho_g \Delta\rho g}{\mu^2}\right]^{1/3} = Ar^{1/3} = \left(\frac{3}{4} C_D Re_p^2\right)^{1/3} \quad (4)$$

$$u_t^* = u_t\left[\frac{\rho_g^2}{\mu(\Delta\rho) g}\right]^{1/3} = \frac{Re_p}{Ar^{1/3}} = \left(\frac{4}{3}\frac{Re_p}{C_D}\right)^{1/3} \quad (5)$$

- Figure 10 $(u_t^*, d_p^*) \longrightarrow u_t, d_p$ (p. 81)

- (3) for spherical particles:

$$C_D = \frac{24}{Re_p} + 3.3643 Re_p^{.3471} + \frac{.4607 Re_p}{Re_p + 2682.5}, \quad \text{for } \phi_s = 1 \quad (6)$$

- fig. 10 for $0.5 < \phi_s < 1$:

$$u_t^* = \left(\frac{18}{d_p^{*2}} + \frac{2.335 - 1.799\phi_s}{d_p^{*0.5}} \right)^{-1}, \quad 0.5 < \phi_s < 1 \qquad (7)$$

- (7) for spherical particles ($\phi_s = 1$):

$$u_t^* = \left(\frac{18}{d_p^{*2}} + \frac{.591}{d_p^{*.5}} \right)^{-1} \qquad (8)$$

- Fig 11 (p. 82) for U_t/U_{mf} vs. $d_{sph}^* = d_{sph} \left(\frac{\rho_g(\rho_s - \rho_g)g}{\mu^2} \right)$

- ex. 3 (p. 82-83)

5) Mapping of fluidization Regimes

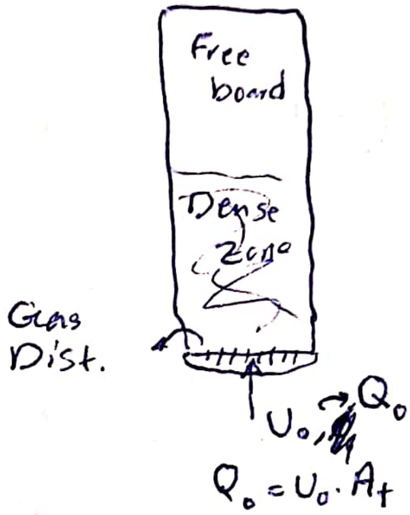
- Fig. 16 (p. 89) → flow regime

- where $U_o^* = U_o \left(\frac{\rho_g}{\mu(\rho_p - \rho_g)g} \right)^{1/3}$

- ex. 4 (p. 91)

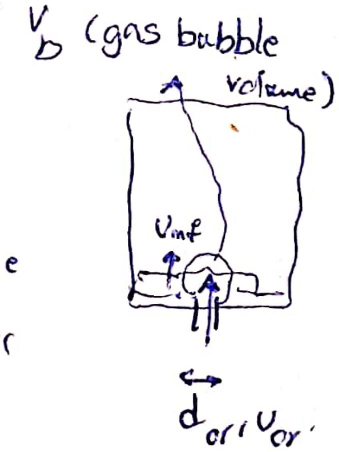
chap. 1 : The Dense Bed: Distributors, Gas jets, and Pumping power

(1) Distributors



- for small scale: Porous material
- Distributor Types → Perforated Plate
  → Multiorifice Plate
- Read "Ideal Distributors" from p. 95
- also "Perforated or Multiorifice plates" and "tuyeres and caps"

(2) Gas Entry Region of a bed



Above a single orifice with background flow

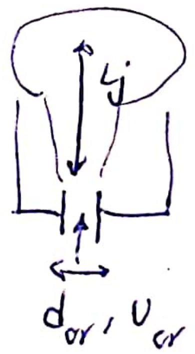
- Davidson & Schüler:

V_{or} → volumetric gas flow rate through a single orifice

$$V_b = 1.138 \frac{V_{or}^{6/5}}{g^{3/5}}$$

- Fig. 5.6 p. (99)

(3) Gas jets in FB



- Gas jet penetration length (L_j)

$$\frac{L_j}{d_{or}} = 21.2 \left(\frac{U_{or}^2}{g d_{or}}\right)^{0.37} \cdot \left(\frac{d_{or} U_{or} \rho_g}{\mu}\right)^{.05} \left(\frac{\rho_g}{\rho_s}\right)^{.68} \left(\frac{d_p}{d_{or}}\right)^{.24}$$

(4) ΔP requirements across distributors

- $\Delta P_d = [.2 - .4]\, \Delta P_b$ (normally) → $\Delta P_d \cong .3\, \Delta P_{bed}$

where: $$\frac{\Delta P_b}{L_{mf}} = (1-\varepsilon_{mf})(\rho_p - \rho_g)\frac{g}{g_c} \quad @\ mf$$

$\Delta P_d \uparrow \longrightarrow$ power input $\uparrow \Rightarrow \Delta P_t \cong \Delta P_d + \Delta P_d \; (+\Delta P_{fitting} + \dots)$

- $\Delta P_d \propto \begin{cases} U_0 & \text{for porous plate} \\ U_0^2 & \text{for perforated plate} \end{cases}$

5) Design of Gas Distributors

- Procedure for perforated plate gas distributors:

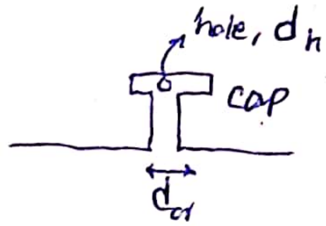
1. Determine the necessary ΔP_d ($\approx .3 \Delta P_b$)
2. Calculate $Re_t = \frac{\rho_g U_0 d_t}{\mu}$ $\quad d_t$ = diameter of tower
3. $C_{d,or}$

Re_t	100	300	500	1000	2000	>3000
C_d	.68	.7	.68	.64	.61	.60

4. $U_{or} = C_{d,or} \left(\frac{2\Delta P_d}{\rho_g}\right)^{1/2}$
5. $\frac{U_0}{U_{or}} < .1$ (normally)
6. $N_{or} \equiv$ No. of orifices per unit area

$$U_0 \times 1 = \underset{?}{N_{or}} \times \left(U_{or} \cdot \frac{\pi d_{or}^2}{4}\right) \; ?$$

7. For a tuyere type with an inlet orifice:



$$N_{or} = \left(\frac{\text{No. of tuyeres}}{\text{per Area}}\right) \times \left(\frac{\text{No. of holes}}{\text{per tuyere}}\right)$$

- Agitating Distributors

$$\alpha = \frac{\rho_g U_{or}^2 / 2g_c}{\Delta P_b} \equiv \frac{\text{Kinetic energy of orifice jets}}{\text{Resistance of the bed}}$$

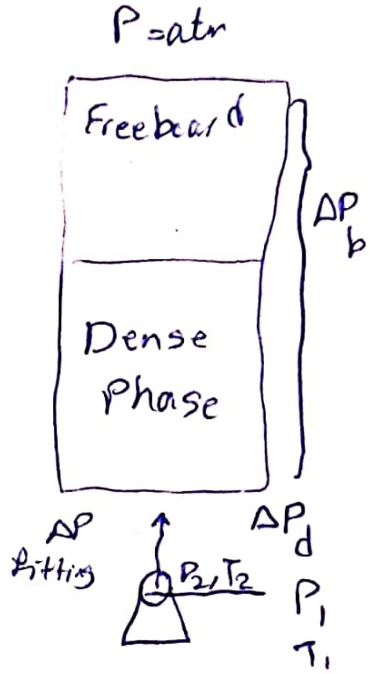
If $\alpha > 1 \longrightarrow$ channeling, gas bypassing

If $\alpha \ll 1 \longrightarrow$ gas jet doesn't play an important role in the mixing

- ex 1 (p. 106-107)
- ex 2 (p. 108)

6 Power Consumption

$$w_{s,ideal} = -\int_{P_1}^{P_2} \hat{v}\, dP \quad (J/kg) \qquad (1)$$



$\dot{V}$ = volumetric gas flow rate

$PV = nRT$, $P\hat{V} = RT$

$$\Rightarrow \dot{w}_{s,ideal} = \frac{\gamma}{\gamma-1} P_1 \dot{V}_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \qquad (2)$$

where: $\gamma = \frac{C_p}{C_v}$, $R = C_p - C_v$ (3)

- If comp. Adiabatic Reversible : $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ (4)

where $\gamma \cong \begin{cases} 1.67 & \text{for monoatomic gases} \\ 1.4 & \text{" diatomic "} \\ 1.33 & \text{" triatomic "} \end{cases}$ (5)

$$-\dot{w}_{actual} = \frac{\dot{w}_{ideal}}{\eta_{comp}} \quad (6) \qquad \Rightarrow T_{2,actual} = T_1 \left[1 + \frac{1}{\eta} \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right] \quad (7)$$

η_{comp} → Comp. efficiency

- if $\eta \to 1 \rightarrow T_{2,actual} = T_2$

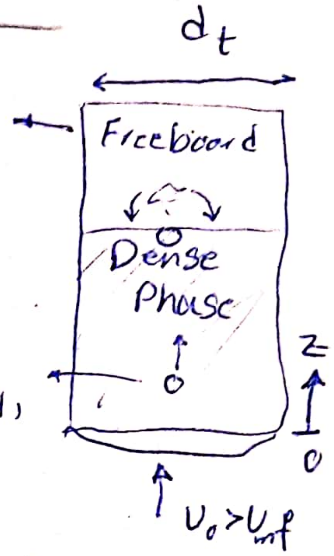
- ex 3 . (p. 110-111)

chap. 5: Bubbles in Dense Beds

(1) Rise Rate of Bubbles

1.1 Single Rising bubbles

G-S contactor



- shape $\sim$ { spherical for small d_b ; X $\sim$ large d_b
- Rising velocity { ↓ small d_b ; ↑ large d_b
- Coalescence of bubbles both for G-L & G-S contactors + Break up
- Flow field interaction: wall effect
- Rising velocity of gas bubbles (for no wall effect)
  - In a G-L contactor [Taylor & Davies]: $u_{br} = \frac{2}{3}(gR_n)^{1/2}$ (1) — with respect to emulsion phase
  - Emperical [Davidson] for G-S contactors: $u_{br} = .711\sqrt{gd_b}$ (2)

  ↳ both can be valid for $\frac{d_b}{d_t} < .125$
- considering wall effect:

$$u_{br} = [.711\sqrt{gd_b}] \times 1.2\exp\left(-1.49\frac{d_b}{d_t}\right) \quad (3)$$

↳ for $\frac{d_b}{d_t} < .6$

- If $d_b/d_t > 60\%$ ⟹ slugging
- $U_o > U_{mf}$ ⟹ $U_o - U_{mf}$ ⟹ gas bubbles ⟹ Emulsion phase @ ε_{mf}, U_{mf}

$$U_g|_{emulsion} = \frac{U_{mf}}{\varepsilon_{mf}}$$

G-L & G-S contactors:

1. Transfer of gas into or out of gas bubbles
2. Presence of solids in gas bubbles

- "The analogy between bubbles" - [Essentials of fluidization] p. 150

1.2 Davidson Model for gas flow at bubbles

Basis of Davidson Model:

1. Irrotational fluid flow: $\vec{\omega}=0$, $\vec{\omega}=\frac{1}{2}\vec{\nabla}\times\vec{\omega}$ (1)

- Inviscid fluid flow.
- 2D fluid flow + incompressible fluid
- cont. eq. $\Longrightarrow$ $\vec{\nabla}\cdot\vec{u}=0$, $\frac{\partial u_y}{\partial x}+\frac{\partial u_y}{\partial y}=0$ (2) , $\vec{u}=\vec{u}(\psi,\vec{x})$ (3)

. mapping from $\vec{u}\longrightarrow\mathbb{R}$, $\psi(x,y)$

Therefore $\begin{cases} u_x=-\frac{\partial\psi}{\partial y} \\ u_y=+\frac{\partial\psi}{\partial x}\end{cases}$ (4) $\Longrightarrow \frac{\partial}{\partial x}\left(-\frac{\partial\psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right)=0$

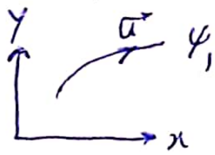
$\Longrightarrow -\frac{\partial^2\psi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y\partial x}=0 \Rightarrow 0=0$ ✓

$\Longrightarrow \psi=\psi(x,y)$ (5) "analytic"

. then: $d\psi=\frac{\partial\psi}{\partial x}dx+\frac{\partial\psi}{\partial y}dy$ (6)

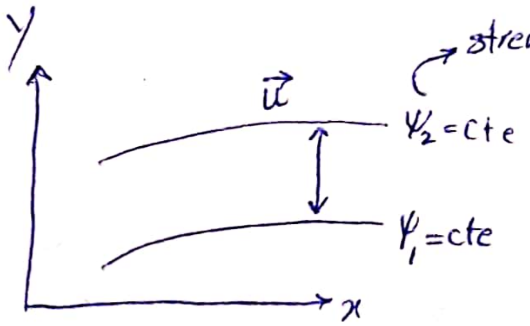
(4) (6) $\Longrightarrow$ $d\psi=u_y dx-u_x dy$ (7) $\Big\}\Longrightarrow$ $0=u_y dx-u_x dy$ (8)

. if $\psi=cte$



$\Longrightarrow u_x dy=u_y dx$

$\Longrightarrow$ (slope) $\left.\frac{dy}{dx}\right|_{\psi=cte}=\frac{u_y}{u_x}=\tan\alpha$



$U_x=-\frac{\partial\psi}{\partial y}\Longrightarrow -d\psi=u_x dy$

$\Longrightarrow \psi_1-\psi_2=\int_{y_1}^{y_2}u_x dy=\dot{V}$

. stream lines: $\vec{u}\times\vec{dr}=0 \Longrightarrow \frac{dx}{u_x}=\frac{dy}{u_y}$ for Cartesian coordinate system

- Vorticity vector ($\vec{\omega}$): $\vec{\omega}=\frac{1}{2}\vec{\nabla}\wedge\vec{u}=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ u_x & u_y & u_z\end{vmatrix}=\omega_x\hat{i}+\omega_y\hat{j}+\omega_z\hat{k}$

$$\Longrightarrow \vec{\omega}=\frac{1}{2}\left\{\underbrace{\left(\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right)}_{\omega_x}\hat{i}+\underbrace{\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right)}_{\omega_y}\hat{j}+\underbrace{\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right)}_{\omega_z}\hat{k}\right\}$$

- In addition, the fluid flow to be Irrotational, then:

$$\omega_z = 0 \Rightarrow \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \Rightarrow \underline{\nabla^2 \psi = 0}$$

2D Fluid flow (Incomp.)

1. $\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$

2. x-comp.: $u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2}\right] + g_x$

3. y-comp. $u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} = \frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left[\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2}\right] + g_y$

- unknowns = $\{u_x, u_y, P\}$

If Incomp. irrotational ($\vec{\omega} = 0$) $\Rightarrow \nabla^2 \psi = 0 \xrightarrow{B.C.s} \psi = \psi(x,y)$

$$\Rightarrow \begin{cases} u_x = -\frac{\partial \psi}{\partial y} \\ u_y = \frac{\partial \psi}{\partial x} \end{cases} \Rightarrow \begin{cases} u_x = u_x(x,y) \\ u_y = u_y(x,y) \end{cases}$$

- finally $P = ?$: stokes: $u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \frac{-1}{\rho}\frac{\partial P}{\partial x} + g_x \Rightarrow P = P(x,y)$

2D fluid flow & Irrotational

- $\omega_z = 0 \Rightarrow 0 = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 0$ (1) $\Rightarrow \vec{u} \rightsquigarrow \phi(x,y)$

"Analytic multivariable function"

- mapping $\longrightarrow$ functional $\vec{u} \longrightarrow \mathbb{R}$

, then $\begin{cases} u_x = -\frac{\partial \phi}{\partial x} \\ u_y = -\frac{\partial \phi}{\partial y} \end{cases}$ (2)

- then: $\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right) = 0 \Rightarrow -\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x} = 0 \Rightarrow 0 = 0$ (3)

(2) $\Rightarrow \left.\frac{dy}{dx}\right|_{\phi = cte} = \frac{-u_x}{u_y}$ (4) $\nRightarrow$ slope ϕ = slope $\vec{u}$

- Moreover: Incomp. fluid flow $\Rightarrow \vec{\nabla}\cdot\vec{u} = 0$

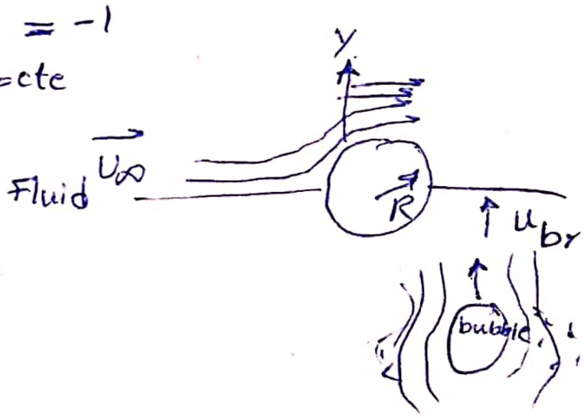
$$\rightarrow \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) = 0$$

$$\Rightarrow \nabla^2 \phi = 0 \quad (7)$$ "Laplace eq." $\phi \equiv$ Velocity Potential

- If $\phi = cte \rightarrow d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = -u_x dx - u_y dy$

$$\xrightarrow{d\phi = 0} u_y dy = -u_x dx \rightarrow \left.\frac{dy}{dx}\right|_{\phi = cte} = -\frac{u_x}{u_y} \quad (8)$$

- orthogonal fields $\Rightarrow$ $\begin{cases} \psi \cdot \phi = 0 \\ slope|_{\psi = cte} \cdot slope|_{\phi = cte} = -1 \end{cases}$



Davidson Model: Assumptions made:

1. Gas bubble: $\longrightarrow$ solid free
2. Spherical bubble 3D, cylinder in 2D
3. Solids around the bubble $\begin{cases} \text{Inviscid fluid flow } (\rho_s(1-\varepsilon_{mf}) \neq \rho_s) \\ \text{emulsion phase @ } \varepsilon = \varepsilon_{mf} \end{cases}$ (1)
4. Gas flow inside the emulsion phase: incomp. fluid flow. $\vec{u_s} \downarrow$ porous media ε_{mf} $\vec{u_g}$

flow of gas in porous medium: Darcy's eq.:

$$\vec{u_g} - \vec{u_s} = -k \vec{\nabla P} \quad (2)$$

5. Far from the gas bubble: $\frac{\Delta P_b}{L_{mf}} = (1-\varepsilon_{mf})(\rho_s - \rho_g)\frac{g}{g_c}$ (3)
6. constant Pressure @ gas bubble: $\nabla^2 \phi = 0$ (4)

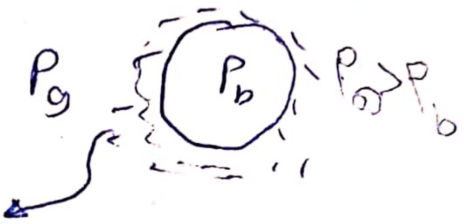
where for solid $\begin{cases} u_r = -\frac{\partial \phi}{\partial r} \\ u_\theta = \frac{-1}{r}\cdot\frac{\partial \phi}{\partial \theta} \end{cases}$, $\vec{u_s} = u_r \vec{r} + u_\theta \vec{\theta}$ (5)

for solid $\begin{cases} u_r = u_{br}\left(\frac{R^3}{r^3}\right)\cos\theta \\ u_\theta = u_{br}\left(\frac{R^3}{2r^3}\right)\sin\theta \end{cases}$ (6)

- for the gas phase through the porous medium:

$$② \Rightarrow \vec{u}_g = \vec{u}_s - k\vec{\nabla}P_g \quad , \text{where } k = cte \quad ⑦$$

$$\text{Gas} \begin{cases} \vec{\nabla}\cdot\vec{u}_g = 0 \\ \vec{\nabla}\cdot\vec{u}_s = 0 \end{cases} \quad ⑧ \qquad \varepsilon = \varepsilon_{mf}$$



$$⑦ \Rightarrow \vec{\nabla}\cdot\vec{u}_g = \vec{\nabla}\cdot\vec{u}_s - \vec{\nabla}\cdot(k\vec{\nabla}P) \quad ⑨$$

$$⑧, ⑨ \Rightarrow k\vec{\nabla}\cdot\vec{\nabla}P_g = 0 \Rightarrow \nabla^2 P_g = 0 \quad ⑩ \quad \text{(around the gas bubble)}$$

$$\text{B.C.s} \begin{cases} @\ r = R : & P_g = P_b \\ @\ r \to \infty : & \begin{cases} \vec{u}_s = 0 \\ \vec{u}_g = u_{mf} \ @\ \varepsilon_{mf} \end{cases} \quad ⑪ \Rightarrow \dfrac{u_{mf}}{\varepsilon_{mf}} - 0 = -kP_g \end{cases}$$

$$⑩, ⑪ \Rightarrow P_g - P_b = -\underbrace{\frac{U_{mf}}{k\varepsilon_{mf}}}_{u_f \text{ in emulsion phase}}\left(r - \frac{R^3}{r^2}\right)\cos\theta \quad ⑫$$

- Moving coordinate system:

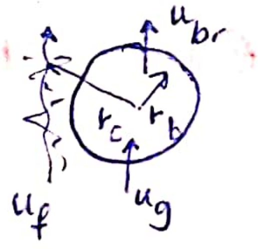
$$\begin{cases} u_r = -U_{br}\left(1 - \dfrac{R^3}{r^3}\right)\cos\theta \\ u_\theta = +U_{br}\left(1 + \dfrac{R^3}{2r^3}\right)\sin\theta \end{cases} \quad ⑬ \quad \text{"For Solid Phase"}$$

- For gas phase

$$\begin{cases} u_{rg} = u_{rs} - k\dfrac{\partial P}{\partial r} \\ u_{\theta g} = u_{\theta s} - \dfrac{k}{r}\dfrac{\partial P}{\partial \theta} \end{cases} \quad ⑭$$

Davidson Model (Stream lines around the gas bubble)

I) $U_{br} < U_f \left(= \dfrac{U_{mf}}{\varepsilon_{mf}}\right)$ → with respect to emulsion phase

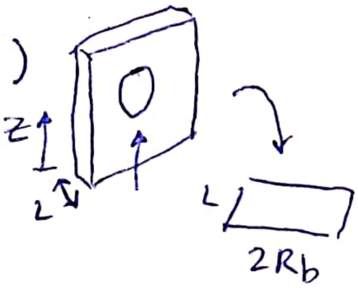


$$\text{cloud} \begin{cases} \dfrac{R_c^2}{R_b^2} = \dfrac{u_{br} + u_f}{u_{br} - u_f} & \text{for 2D} \\ \dfrac{R_c^3}{R_b^3} = \dfrac{u_{br} + 2u_f}{u_{br} - u_f} & \text{for 3D} \end{cases}$$

$$f_c = \frac{V_c}{V_b} \begin{cases} \dfrac{2u_f}{U_{br} - u_f} = \dfrac{2U_{mf}/\varepsilon_{mf}}{u_{br} - U_{mf}/\varepsilon_{mf}} & \text{for 2D} \\[2ex] \dfrac{3u_f}{U_{br} - u_f} = \dfrac{3u_{mf}/\varepsilon_{mf}}{u_{br} - U_{mf}/\varepsilon_{mf}} & \text{for 3D} \end{cases}$$

Flow rate of gas into and out of a bubble

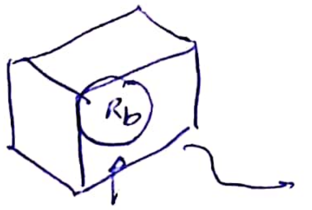
2D)



$$\dot{Q}_g = 4U_{mf} R_b L = 4U_f \varepsilon_{mf} R_b L$$

→ max cross section ⇒ $L \times 2R_b = A_b^{max}$

3D)



$$\dot{Q}_g = 3U_{mf} \pi R_b^2 = 3U_f \varepsilon_{mf} \pi R_b^2$$

max cross section: $A_b^{max} = \pi R_b^2$

$-\dot{Q}_g = u_g A$

$$\Rightarrow u_g \text{ @ } A_b^{max} = \begin{cases} 2D) & u_g = 2U_{mf} \\ 3D) & u_g = 3u_{mf} \end{cases}$$

wake

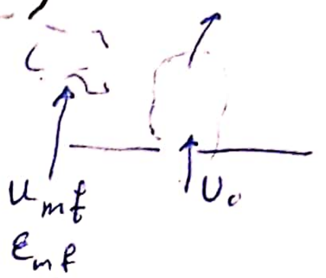
→ wake (V_w) : $f_w = \dfrac{V_w}{V_b}$ → Fig 8 (p. 129)

Solids in void spaces (gas bubbles)

. 2 - 1 Vol 1.

- ex. 1 (p. 126)

② Coalescence and Splitting of Bubbles



$$V_b = 1.138\,\frac{U_{or}^{6/5}}{g^{3/5}} \quad [m^3] \quad ①$$

$$n_b = \frac{U_{or}}{V_b} = \frac{54.8}{U_o^{1/5}} \quad [s^{-1}] \quad ②$$

where $\begin{cases} U_{or} \equiv \text{Volumetric gas flow rate through an orifice} \\ V_b \equiv \text{Gas bubble volume} \end{cases}$

– $U_{or} = 200 - 2000 \ \frac{cm^3}{s} \xRightarrow{②} n_b = 19 - 12 \ [s^{-1}]$

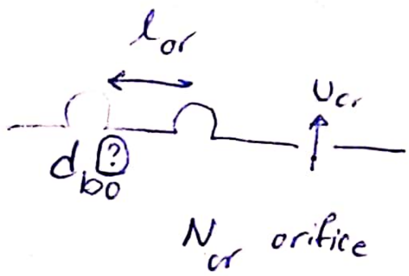
n_b (experimental) $\approx 7 \ [s^{-1}]$

– Starting Point of coalescence

if $d < 3R_b$

③ Bubble Formation above a Distributor

I) $U_o > U_{mf}$



$U_o - U_{mf} \longrightarrow$ gas bubbles, $U_f = \frac{U_{mf}}{\varepsilon_{mf}}$

$(U_o - U_{mf}) = V_{or} \cdot N_{or}$ ①

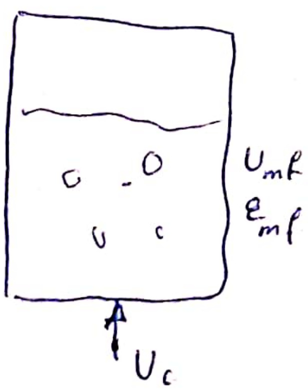
$\downarrow$

volumetric gas flow rate through an orifice

. no coalescence: $d_{bo} < l_{or}$

using $V_b = 1.138\,\frac{U_{or}^{6/5}}{g^{3/5}} \Rightarrow d_{bo} = 1.3\,\frac{U_{or}^{.4}}{g^{.2}}$ ②

"above a single orifice"



$$①, ② \Longrightarrow d_{bo} = \frac{1.3}{g^{.2}}\left(\frac{U_o - U_{mf}}{N_{or}}\right)^{.4} \quad ③ \quad (\text{for } d_{bo} < l_{or})$$

- orifice arrangements

$\left\{\begin{array}{ll} \updownarrow l_{or} : & N_{or} = \dfrac{1}{l_{or}^2} \quad \text{"square array of orifices"} \quad (4) \\ \downarrow l_{or} : & N_{or} = \dfrac{2}{\sqrt{3}\, l_{or}^2} \quad \text{"equi-lateral triangle array of holes"} \end{array}\right.$

II) High gas flow rate

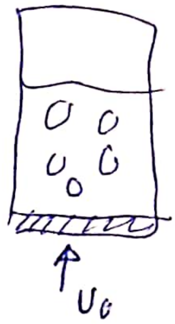
- for equi-lateral triangle array of orifices:

$$N'_{or} = \frac{2}{\sqrt{3}\, d_{bo}^2} \Longleftrightarrow d_{bo}^2 = \frac{2}{\sqrt{3}\, N'_{or}} \xRightarrow{HW} d_{bo} = \frac{2.78}{g}(U_o - U_{mf})^2$$

$l_{or} = d_{bo}$

III) Porous Plate Gas Distributor

with $d_{bo} > l_{or} \Longrightarrow d_{bo} = \dfrac{2.78}{\underset{\hookrightarrow 980}{g}}(\overset{cm/s}{U_o - U_{mf}})^2$ [cm]



- ex 2 (p. 132)

+ Slug flow

chap. 6 : Bubbling Fluidized Beds

(1) Experimental Findings

- $U_o - U_{mf} \xrightarrow{gas}$ bubble formation
- Emulsion phase @ stagnant $(u_p = 0)$ $\begin{cases} \varepsilon_{mf} \\ U_{mf} \longrightarrow U_f = \dfrac{U_{mf}}{\varepsilon_{mf}} \end{cases}$

1.1 on G-A & G-B

~~for dp > 30~~ μm

- The Bubble gas is not given by $(U_o - U_{mf})$
- The emulsion ~~phase is not~~ voidage (ε_e) doesn't stay @ (ε_{mf}) as $U_o > U_{mf}$
- The emulsion phase is not essentially stagnant and movement (flow patterns) and gulf streaming.

for solids G-A & ~~G-B~~ AB :

$$\left(\frac{\varepsilon_e}{\varepsilon_{mf}}\right)^3 \left(\frac{1-\varepsilon_{mf}}{1-\varepsilon_e}\right) = \left(\frac{U_e}{U_{mf}}\right)^{.7}$$

$\{\varepsilon_{mf}, U_{mf}, \varepsilon_e, U_e\} \longrightarrow$ 3 unknowns

for G-B & G-D:

- Experimental findings $\Longrightarrow$ $U_e = U_e(z)$ & $U_e > U_{mf}$
- $$\frac{U_e - U_{mf}}{U_o - U_{mf}} = \begin{cases} \cancel{1/8}\ 1/3 & \text{for 3D FB} \\ 1/8 & \text{for 2D FB} \end{cases}$$

(2) Estimation of Bed Properties

$$\psi \equiv \left[\frac{\text{observed bubble flow}}{\text{excess flow}}\right] = \frac{v_b}{(U_o - U_{mf}) A_t}$$

where $\begin{cases} \psi \equiv \text{Bubble gas flow} \\ v_b \equiv \text{volumetric gas flow rate through gas bubble.} \end{cases}$

- Fig. 6 (p. 143) for $\psi = \psi(z/d_t)$

- up to $z/d_t \cong 1 \Rightarrow \psi = \begin{cases} .8 & \text{for G-A} \\ .65 & \text{for G-B} \\ .25 & \text{for G-D} \end{cases}$

2.2 Bubble Size & Growth

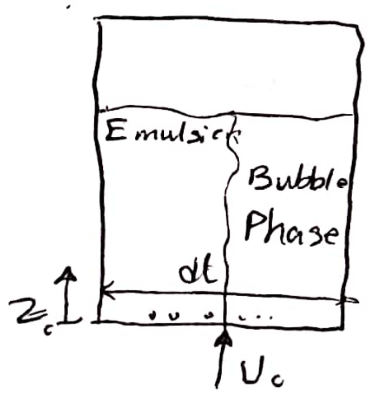
$$\bar{d}_b = \frac{\sum n_{bi} d_{bi}}{\sum n_{bi}} \quad (1)$$

$$\bar{d}_b^2 = \frac{\sum n_{bi} d_{bi}^2}{\sum n_{bi}} \quad (2)$$

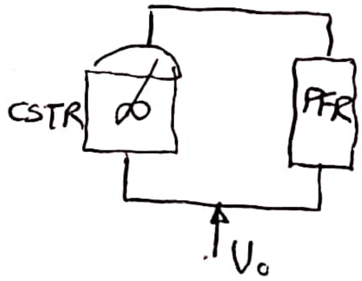
$$\bar{d}_b^3 = \frac{\sum n_{bi} d_{bi}^3}{\sum n_{bi}} \quad (3)$$

$$\bar{d}_b = \frac{\sum n_{bi} d_{bi}^3}{\sum n_{bi} d_{bi}^2} \quad (4)$$ Sauter Mean Diameter

Flow Model



$\bar{d}_b \rightarrow \bar{U}_b \Rightarrow$ Fractional Conversion (X_A)



Bubble Size Correlations ($d_b = d_b(z)$)

1. Wen et al. for G-B & G-D:

$$\frac{d_{bm} - \bar{d}_b}{d_{bm} - d_{bo}} = \exp\left(-\frac{.3z}{d_t}\right) \quad (1)$$

where: $d_{bo} \equiv$ Initial bubble diameter $\Rightarrow d_{bo} = 1.3 \frac{V_{or}^{.4}}{g^{.2}}$ [cm]

$$d_{bo} = \frac{1.3}{g^{.2}} \left(\frac{U_0 - U_{mf}}{N_{or}}\right)^{.4} \text{ [cm] for } d_{bo} < l_{or}$$

$$d_{bo} = \frac{2.78}{g} (U_0 - U_{mf})^2 \text{ [cm] for } d_{bo} > l_{or}$$

$d_{bm} \equiv$ The limitting size of gas bubble expected in a deep bed

$$\Rightarrow d_{bm} = .65 \left[\frac{\pi}{4} d_t^2 (U_0 - U_{mf})\right]^{.4}$$

2. Werther proposed the following correlations for G-B & Porous plate:

$$\begin{cases} d_b = .853\,[1+.272(U_0-U_{mf})]^{1/3}\,[1+.0684z]^{1.21} & [cm] \\ d_t > 20\ cm, \quad 1 < U_{mf} < 8\ cm/s, \quad 100 < d_p < 350\ \mu m \\ 5 < U_0 - U_{mf} \leq 30\ cm/s \end{cases}$$

$\boxed{2.3}$ Bubble Rise Velocity

- for a single bubble: $U_{br} = .711\sqrt{g d_b}$ → with respect to the emulsion phase

$\Longrightarrow U_b = (U_0 - U_{mf}) + U_{br}$ [Not real] → for bubbles in a bubbling bed

- for G-A & G-D $\Longrightarrow U_b = U_b(d_t, d_p)$

+ Werther et. al. $\Longrightarrow U_b = \psi(U_0 - U_{mf}) + \alpha U_{br}$, $\psi = \dfrac{V_b}{(U_0-U_{mf})A_t}$

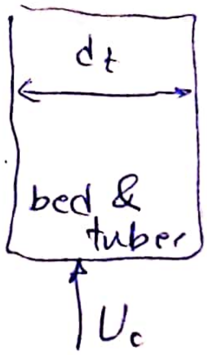
Geldart	A	B	D
α	$3.2d_t^{1/2}$	$2d_t^{1/2}$	.87
d_t(m)	.05 - 1	.1 - 1.0	.10 - 1.0

+ Werther et al.
  - → G-A & $d_t \leq 1^m$: $U_b = 1.55[(U_0-U_{mf})+14.1(d_b+.005)]d_t^{.32} + U_{br}$
  - → G-B & $d_t \ll 1^m$: $U_b = 1.6[(U_0-U_{mf})+1.13d_b^{.5}]d_t^{1.35} + U_{br}$

$\boxed{2.4}$ Beds with Internals

- $d_t \longrightarrow d_{te}$

$$d_{te} = \frac{4[\text{cross-sectional area available}]}{\text{total wetted perimeter of bed \& tubes}}$$



- ex. 1 (p. 150)
- ex. 2 (p. 151)

3) Scale-up and Scale-Down

I) Horio et al. 1. $\frac{U_o - U_{mf}}{\sqrt{g d_p}}$ 2. $\frac{U_{mf}}{\sqrt{g d_p}}$

II) Fitzgerald et al. 1. $Re_p = \frac{\rho_g U_o d_p}{\mu}$ 2. Density ratio $= \frac{\rho_s}{\rho_g}$

3. $Fr = \frac{U_o}{\sqrt{g d_p}}$

4. Geometric similarity of distributor, bed, & particle $= \frac{L}{d_p}$

Fitzgerald procedure proposed is as follows:

1. Calculate $(\frac{\rho_s}{\rho_g})_1$ for the system to be modeled.
2. choose a gas for the model (Ambient air) $\Rightarrow \rho_{g_2} = cte$

$$(\frac{\rho_s}{\rho_g})_1 = (\frac{\rho_s}{\rho_g})_{2(model)} \Rightarrow (\rho_s)_{model} = \rho_{g_2} \times (\frac{\rho_s}{\rho_g})_1 \equiv fixed$$

3. Combining $Re_1, Re_2, Fr_1, Fr_2 \Rightarrow \frac{L_2}{L_1} = m = [\frac{\rho_{g_1} \mu_2}{\rho_{g_2} \mu_1}]^{2/3}$
4. $\frac{U_2}{U_1} = \frac{t_2}{t_1} = [\frac{L_2}{L_1}]^{1/2} = \sqrt{m}$

– ex. 3 (p 153-154)

4) Flow Model

4.1 General Interrelationship among Bed Properties

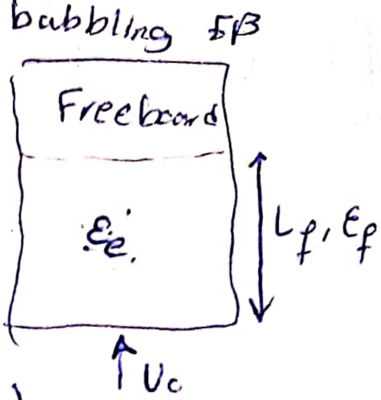
– Mass balance for solids:

$$L_m(1-\varepsilon_m) = L_{mf}(1-\varepsilon_{mf}) = L_{mb}(1-\varepsilon_{mb}) = L_f(1-\varepsilon_f) \quad (1)$$

fixed-bed, min fluidization, min bubbling, bubbling bed

– $\varepsilon_b \simeq 1 \Rightarrow \varepsilon_s|_{bubbles} \cong 0$

$\delta \equiv$ fraction of bubbles in bed
$\varepsilon_f \equiv$ average voidage ②
$\varepsilon_e \equiv$ emulsion 〃



$\Rightarrow \quad \varepsilon_f = \delta + (1-\delta)\,\varepsilon_e$ ③

$\Rightarrow \quad 1-\varepsilon_f = 1-\delta-(1-\delta)\varepsilon_e$ ④ $\Rightarrow \quad 1-\varepsilon_f = (1-\delta)(1-\varepsilon_e)$ ⑤

- If ε_f & δ are known from the experiment, then ε_e can be evaluated using ⑤. However, if ε_f & δ were unknown, then:

lets take $\begin{cases} \varepsilon_e \cong \varepsilon_{mb} & \text{for G-A solids} \\ \varepsilon_e \cong \varepsilon_{mf} & \text{for G-B \& G-D solids} \end{cases}$

4.2 The Simple Two-Phase Model by Toomey et al.

- Basis of the model:

1. $(U_o - U_{mf}) \Rightarrow$ bubble phase
   Emulsion phase @ ε_{mf} & stagnant
2. Rise velocity of bubbles: $u_{br} = .711\sqrt{g d_b}$ ①
3. 〃 〃 〃 emulsion gas: $u_e = U_{mf}/\varepsilon_{mf}$ ② → superficial rise velocity of emulsion gas
4. 〃 〃 〃 solids: $u_s = U_{s,up} = U_{s,down} = 0$ ③
5. Fraction of bed in gas bubbles:

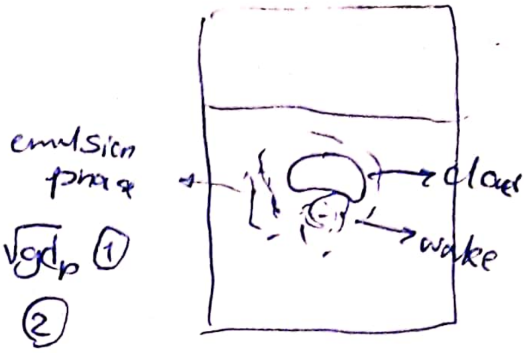
$$U_o A_t = U_{mf} A_t (1-\delta) + U_b A_t \delta \quad ④ \quad \Rightarrow \quad \delta = \frac{U_o - U_{mf}}{U_b - U_{mf}} \quad ⑤$$

6. Fraction of bed emulsion phase:

$$1-\delta = 1-\frac{U_o - U_{mf}}{U_b - U_{mf}} \quad \Rightarrow \quad 1-\delta = \frac{U_b - U_o}{U_b - U_{mf}} \quad ⑥$$

4.3 K–L Model

– for solids of G-A & G-AB :



1. Bubble rise velocity:
   - single bubble: $U_{br} = .711\sqrt{g d_p}$ (1)
   - $U_b = (U_o - U_{mf}) + U_{br}$ (2)
   - For large beds, G-A :

$$U_b = 1.55\left[(U_o - U_{mf}) + 14.1(d_b + .005)\right] d_t^{.32} + U_{br} \quad (3)$$

   - For solids G-B:

$$U_b = 1.6\left[(U_o - U_{mf}) + 1.13 d_b^{.5}\right] d_t^{1.35} + U_{br} \quad (4)$$

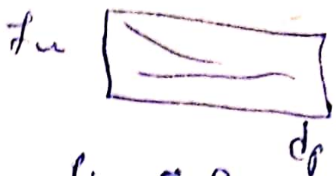
   - Alternatively: $U_b = \psi(U_o - U_{mf}) + \alpha U_{br}$ (5) for G-A, B, D

2. Fraction of the bed in bubbles
   - For slow bubbles ($U_b < U_e$): $\delta = \dfrac{U_o - U_{mf}}{U_b + 2U_{mf}}$ (6)
   - For intermediate bubbles with thick clouds $\left(\dfrac{U_{mf}}{\varepsilon_{mf}} < U_b < 5\dfrac{U_{mf}}{\varepsilon_{mf}}\right)$ (7) ⇒ 7)

     then $\delta = \begin{cases} \dfrac{U_o - U_{mf}}{U_b + U_{mf}} & \text{when } U_b \cong U_{mf}/\varepsilon_{mf} \quad (8) \\[2ex] \dfrac{U_o - U_{mf}}{U_b} & \text{when } U_b \cong U_{mf}/\varepsilon_{mf} \quad (9) \end{cases}$

   - For fast bubbles $\left(U_b > 5\dfrac{U_{mf}}{\varepsilon_{mf}}\right)$, clouds are thin and: $\delta = \dfrac{U_o - U_{mf}}{U_b - U_{mf}}$ (10)
   - For $U_o \gg U_{mf}$: $\delta \cong U_o / U_b$ (11)

3. Fraction of clouds around bubbles: $f_c = \dfrac{3}{\dfrac{U_{br}\,\varepsilon_{mf}}{U_{mf}} - 1}$ (12)



4. wake volume to bubble volume:

$$f_w = \frac{V_{wake}}{V_b} \quad (13) \Rightarrow \text{using } \underline{\text{fig. 5-8}}$$

5. Fraction of bed in emulsion (not counting bubble wake):

$$f_e = 1 - \delta - f_w \cdot \delta \quad (14)$$

6. Fraction of solids in various regions {bubble, cloud, emulsion}

$$\gamma_b, \gamma_c, \gamma_e \equiv \frac{\text{Volume of solids dispersed in b, c, and e, respectively}}{\text{volume of bubble}} \quad (15)$$

$$\Rightarrow \delta(\gamma_b + \gamma_c + \gamma_e) = 1 - \varepsilon_f = (1 - \varepsilon_{mf})(1 - \delta) \quad (16)$$

$$\Rightarrow \gamma_e = \frac{(1 - \varepsilon_{mf})(1 - \delta)}{\delta} - \gamma_b - \gamma_c \quad (17)$$

$$\Rightarrow \gamma_c = (1 - \varepsilon_{mf})(f_c + f_w) = (1 - \varepsilon_{mf})\left[\frac{3}{\frac{U_{br}\,\varepsilon_{mf}}{U_{mf}} - 1} + f_w\right] \quad (18)$$

- From experiments: $\gamma_b \in [.001 - .01]$, generally: $\gamma_b \cong .005$ (19)

7. Rise velocity of wake solids: $U_{s,wake} = U_b$ (20)

8. For G-B solids, Downflow velocity of emulsion solids:

$$U_{s,down} = \frac{f_w \delta U_b}{1 - \delta - f_w \delta} \quad (21)$$

9. Rise velocity of emulsion gas through the bed:

$$U_e = \frac{U_{mf}}{\varepsilon_{mf}} - U_{s,down} \quad (22)$$

- ex.4 (p. 159-160-161)

chap. 11 Solid Mixing & Segregation from "Introduction to Particle Technology" 2nd ed.

I) Introduction, d_p and/or ρ_p

II) Types of solid Mixture (fig 11.1)
- → Perfect solid mixture ✗ (ideal)
- → Random " " ✓
- → Segregated " " ✗

III) Segregation → Difference in d_p, ρ_p, ϕ_s ⇒ segregation

most important factor ↵ ↳ important in fluidization

IV) Mechanisms of segregation (on the basis of d_p)

1. Trajectory segregation

d_p, ρ_p $\xrightarrow[\text{"stokes law"}]{U_o}$ Fluid $\left(\frac{\mu}{\rho}\right)$

$\longmapsto x \longrightarrow$ $U_p = 0$: $x = \dfrac{\rho_p U_o d_p^2}{18\mu}$

2. Percolation of fine Particles

3. Rise of coarse particles on vibration

4. Elutriation Segregation

Reduction of Segregation

1. Δd_p → Segregation ↑ ⇒ $\Delta d_p \approx 0$
2. All solids < 30 μm & $\rho_p \in (2000-3000 \ kg/m^3)$ ⇒ Segregation is not important
   "cohesive" [G-C]
3. Addition of H_2O ⇒ "ordered" or "Interactive"

Mechanisms of Solids Mixing

- Lacey (1954) ⟶ shear Mixing, Diffusive mixing, Convective mixing

Assessing The Mixture

- Quality of a solid mixture

Statistics Relevant to Mixing (Solid Mixture Binary (A,B))

- Mean Composition (MC)
  - ⟶ Actual MC (μ) ⟹ unknown
  - ⟶ an estimation of "μ" ⟹ $\bar{y}$ (using sampling Tech.)
  - ⟶ Suppose that: N samples $\{y_1, y_2, \cdots, y_N\}$

$$\Rightarrow \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

- Standard Deviation & Variance
  - ⟶ actual variance $\equiv \sigma^2$, actual SD $\equiv \sigma$.
    μ (unknown) ⟹ σ^2 & σ (unknown)
  - ⟶ If the actual composition (μ) is known:
    $$\sigma^2 \triangleq \frac{\sum_{i=1}^{N}(y_i - \mu)^2}{N}$$
  - ⟶ If the actual composition (μ) is not known:
    $$\sigma^2 \triangleq \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N-1}$$
  - ⟶ $SD \triangleq \sqrt{\sigma^2}$

- Theoritical Limits of Variance (for a binary Solid Mixture (A,B))
  - ⟶ Upper Limit (completely segregated):
    $$\sigma_{UL}^2 = p(1-p)$$
    p ⟶ weight fraction from sample (n part.)
  - ⟶ Lower Limit (Randomly Mixed):
    $$\sigma_{LL}^2 = \frac{p(1-p)}{n}$$
  - ⟶ $\sigma_{LL}^2 < \sigma_{act.}^2 < \sigma_{UL}^2$

Mixing Indices

- Lacey (1954)
  - → Lacey Mixing index $\triangleq \dfrac{\sigma_{UL}^2 - \sigma^2}{\sigma_{UL}^2 - \sigma_{LL}^2}$
  - → if L.M.I → 0 $\Rightarrow \sigma^2 = \sigma_{UL}^2 \Rightarrow$ completely segregated
  - → If L.M.I → 1 $\Rightarrow \sigma^2 = \sigma_{LL}^2 \Rightarrow$ " random mixture
  - → In reality, L.M.I $\in$ (.75 – 1.)

- Poole et al. (1964)
  - → Poole et al. $\triangleq \dfrac{\sigma}{\sigma_{LL}}$ → better
  - → P.M.I → 1 (for random mixture)

- Tests for percision of mixture composition and variance by T-Student and χ^2 (chi-squared) distribution

① Sample composition
- → N samples $\Rightarrow \bar{y}$ & $S \equiv$ STD
- → $\mu \equiv$ true comp

→ $\mu = \bar{y} \pm \dfrac{t.S}{\sqrt{N}}$

for ex.: @ 95%, N=60 $\Rightarrow t = 2.0 \Rightarrow \mu = \bar{y} \pm .258S$

② Variance
- → N>50
  - → T-test
  - → $\sigma^2 = s^2 \pm t \times \underbrace{E(s^2)}_{= s^2\sqrt{\frac{2}{N}}}$
- → N<50
  - → X Normal $\Rightarrow$ chi-squared
  - → $\sigma_{LL}^2 = \dfrac{s^2(N-1)}{\chi^2_{\alpha}}$
  - → $\sigma_{UL}^2 = \dfrac{s^2(N-1)}{\chi^2_{1-\alpha}}$

where $a \equiv$ significance level $\xrightarrow{\text{for}}$ 90%, $a = .5(1-.9)$
$= .05$

- ex. 1 (p. 330)