Fluidization – Session 1 – Introduction

- Aim / Scope / Objective → Fluidization is one of the more important techniques in chemical engineering processes which has been used wildly in chemical and physical processes. Hence, in this course we study the fundamental basis of fluidized bed (F.B.) reactors and contactors including hydrodynamic and engineering aspects.

- Syllabus
  - (I) Hydrodynamic Basis. 1. Introduction: Fluidization state. 2. Single Particle Suspension. 3. Fluid Flow Through Particle beds. 4. Homogeneous Fluidization. 5. First Equation of change for fluidization. 6. Particle-bed model. 7. 2-phase particle bed model. 8. 2-phase particle bed model predictions. 9. Generalized 2-fluid model.
  - (II) Engineering Basis. 1. Introduction. 2. Fluidization and Mapping of Regimes. 3. Dense Beds: Distributors and Gas jets. 4. Bubbles in Dense beds. 5. Bubbling fluidized bed (F.B.) 6. Entrainment & Elutriation from F.B. 7. High-velocity fluidized beds. 8. Solids mixing and segregation.

- Refrences
  - Fluidization Dynamics. Gibilaro, Butterworth-Heinemann, 2001.
  - " Engineering. Kunii and Levenspiel, " , 1991.
  - Multiphase Flow and Fluidization: Continuum and Kinetic Theory Descriptions. Gidaspow, Academic Press, 1994.
  - Computational Transport Phenomena of Fluid-Particle Systems. H. Arastoopour, D. Gidaspow, and E. Abbasi, Springer, 2017.
  - Principles of Gas-Solid Flows. L.S. Fan and C. Zhu, Cambridge University Press, 1998.

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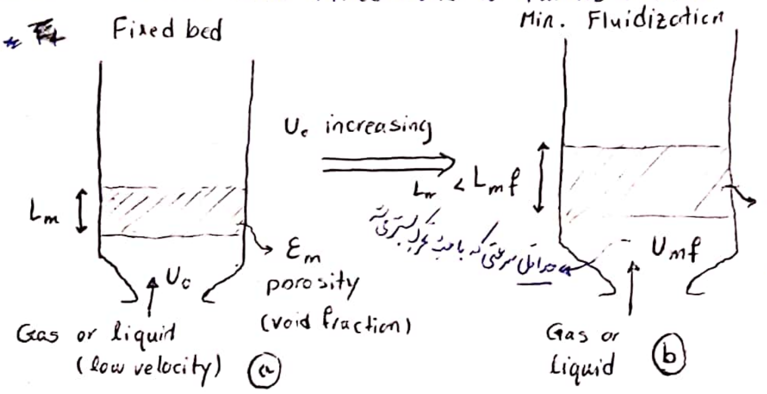
Ch #1 Introduction: Fluidization state

- Fixed Beds
  - ✓ simple mode contacting (operation, Design)
  - we can have G-S, G-S-L, L-S
  - × Performance (Heat & Mass Transport) [Relatively low Heat & Mass coefficients]
  - × Exothermic & Endothermic Chemical Reactions (> 150 kJ/gmol)
    because of non-uniform Temp. Dist. Hence
    - side products
    - Deactivation of Catalyst
    - Hot spot.
    - non-uniform product dist
    - undesired products.
  - × Catalytic Deactivation (need for 2 beds and discontinuity in doing so)
  - × Hot Spot
  - × process control

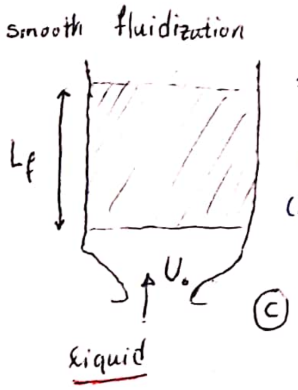
- so we switch from Fixed mode to Fluidized mode



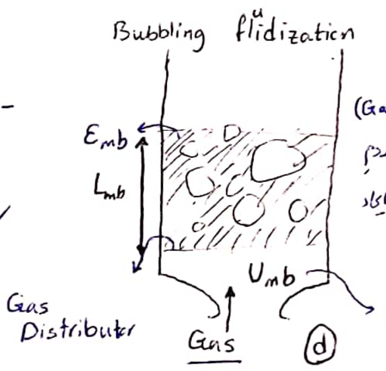
افت فشار سرعتی که باعث حرکت بستر می‌شه

- ذرات از همدیگه کمی فاصله می‌گیرن (بستر منبسط می‌شه)
- ذرات نیروی کمتری بهم وارد می‌کنن
- ولی هیچ ذره‌ای از بستر خارج نمی‌شه
- ذرات معلق می‌شن ولی حرکت زیادی نمی‌تونن بکنن

U_o increasing

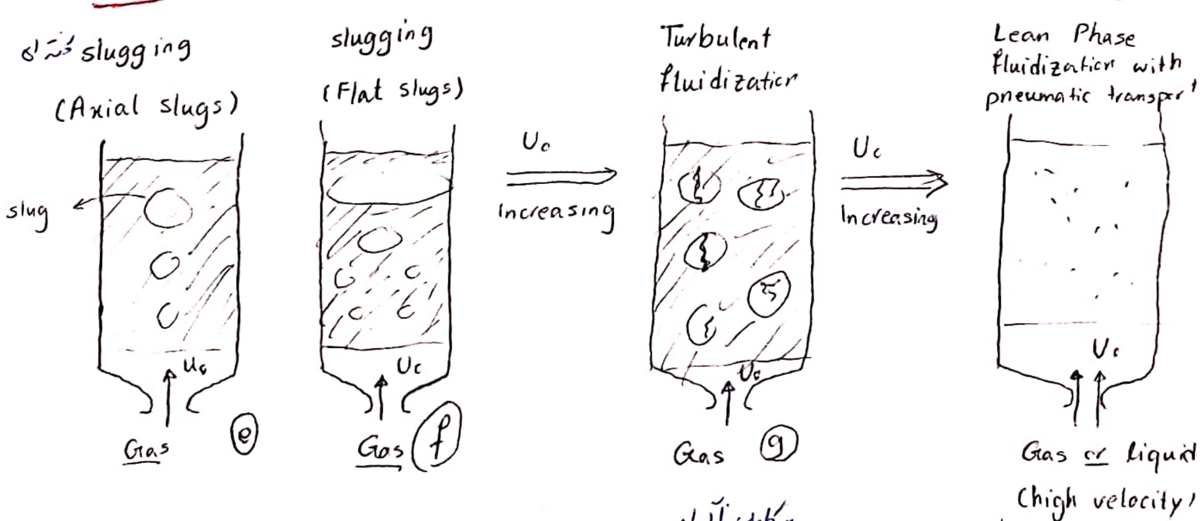


- بستر منبسط‌تر شده و ذرات جامد به صورت همگن توزیع می‌شن؛ یعنی تعداد ذرات توی همه جا یکسانه (Homogeneous)



- یه سری حفره‌ی گازی درین ذرات (Gas bubble) باید ایجاد می‌شه
- این فرایند در اثر حرکات تصادفی و نامنظم ذرات جامد با افزایش سرعت گاز، ایجاد می‌شن
- بستر غیرهمگن
- افت فشار سرعتی که باعث غیرهمگنی بستر می‌شه
- این بستر ناپایدار شده

U_o increasing



- بستر کاملی آرامه

- جریان سیال ذرات را با خودش حمل می‌کنه و می‌بره
- حمل بادی ذرات
- بستر به موقعیت می‌تونه مفید یا مضر باشه

- برای جلوگیری از فرار ذرات خارج از بستر، از سیکلون ها استفاده می کنیم
  - توی بستر باشه. Internal cyclone
  - بیرون باشه. External (این طراحی باعث ... Choking می‌شه) (امکان ...)

- یه مدل دیگه بسترهای فواره‌ای (spouted bed) که دیگه gas distributor ندارن

- رفتار سیال‌گونه‌ی این بسترهای سیال
  - یعنی یک تکه چوب هم بندازی روش شناور می‌شه (light object floats)
  - اگه توی دیواره‌ی بستر سوراخ ایجاد کنیم، به شکل جت از آن ذرات سیال بیرون می‌زنه
  - ...

- Characteristics of Gas fluidized beds
  - → Bed behaves like liquid of the same bulk density - can add or remove particles
  - → Rapid particle motion - good solids mixing
  - → very large surface area available

- Advantages
  - → Good G-S mass transfer in dense phase
  - → Good heat transfer
  - → Easy solids handling
  - → Low pressure drop

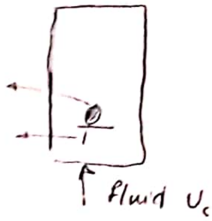
- Disadvantages
  - → Bypass of gas in bubbles
  - → Broad RTD gas and solids
  - → Erosion of internals
  - → سایش Attrition of solid
  - → Difficult scale-up

- Significanse of Fluidized Beds
  - → Advanced Materials
    - → Silicon Production for Semiconductor and Solar Industry
    - → Coated Nanoparticles
    - → Nano Carbon Tubes
  - → Chemical & Petrochemical
    - → Cracking of hydrocarbons
    - → Gas Phase Polymeric Reactions
  - → Combustion / Pyrolysis
    - → Combustion / Gasification of Coal
    - → Pyrolysis of Wood Waste
    - → Chemical Looping Combustion
  - → Physical Operations
    - → Coating of Metals and Glass Objects.
    - → Drying of Solids
    - → Roasting of food
    - → Classify Particles
  - → Pharmaceutical
    - → Coating of Pills
    - → Granulation
    - → Production of Plant & Animal cells

Single Particle ~~Settling Velocity~~ Suspension

- if $U_0 = U_t$ (critical velocity) (without any support) $\Rightarrow \sum \vec{F}_p = 0$
- if $U_0 > U_t \Rightarrow \sum \vec{F}_p > 0$, $\sum \vec{F}_p = m_p \frac{dU_p}{dt}$

slip velocity $= U_0 - U_p$ or $U_f - U_p$



$U_0 \uparrow \Rightarrow$ slip vel. $\downarrow \Rightarrow$ drag force $\downarrow$ $(F_D \propto U_{slip})$ $\xrightarrow{\text{until}}$ $U_0 - U_p = U_t$ or $U_p = U_0 - U_t$ $(\sum \vec{F}_p = 0)$ (که ذره این سرعت ثابت رو داره)

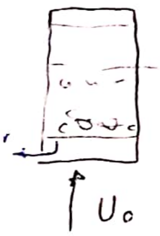
Multi Particle Suspension

- $U_0 \uparrow \rightarrow U_{critical}$ (U_{mf}) $\Rightarrow$ Bed weight = Interaction force

$W = F_D |_{Particles}$

- $U_0 \uparrow \rightarrow$ depends on the fluid (gas or liquid)

Bubbling fluidization ⌟ ⌞ Homogeneous (Smooth) fluidization

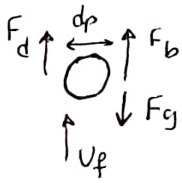


Settling Velocity of a Single Particle (Unhindered : بدون مزاحمت از دیگر ذرات)

$\sum \vec{F}_p = 0 \Rightarrow \vec{F}_{int.} + \vec{F}_g = 0 \quad (\vec{a}_p = 0)$, drag coefficient $C_D = \frac{F_D/A_p}{\frac{1}{2}\rho u_f^2}$ (A_p: projected area)

eq. state

$F_{int.}$: dependent of flow regime (Re_p) ↳ $C_D(Re_p)$



Regime 1. Creeping flow Regime ($Re_p < .1$)

$$Re_p = \frac{\rho_f u_f d_p}{\mu_f} < .1 \quad , \quad F_{int.} = F_b + F_d = \underbrace{V_p \rho_f g}_{\pi d_p^3/6} + 3\pi d_p \mu_f u_f$$

Total drag force

$$F_{int.} = F_g \Rightarrow \frac{\pi d_p^3}{6}\rho_f g + 3\pi d_p \mu_f u_f = \frac{\pi d_p^3}{6}\rho_p g \quad \text{(spherical particle)}$$

$$\Rightarrow C_D = \frac{F_D/\frac{\pi d_p^2}{4}}{\frac{1}{2}\rho_f u_f^2} = \frac{3\pi d_p \mu_f u_f}{\frac{1}{2}\rho_f u_f^2 \cdot \frac{\pi d_p^2}{4}} = \boxed{\frac{24}{Re_p}} \Rightarrow \boxed{U_t = \frac{(\rho_p - \rho_f) g d_p^2}{18\mu_f}}$$

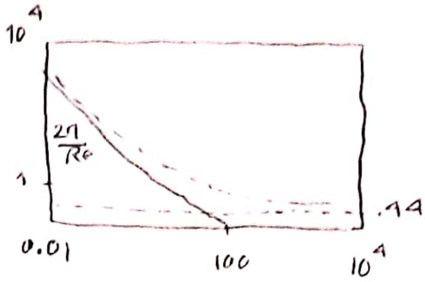
Regime 2. Inertial Flow Regime ($Re_p > 500$)

$$C_D \cong .44 = \text{cte} \Rightarrow F_D \checkmark \Rightarrow u_t = \checkmark$$

HW-99-12-3-1 (Name) Derive u_t for inertial flow regime.

Regime 3. All flow regimes "Dallovalle" [1948]

$$C_D = [0.63 + 4.8\, Re_p^{-0.5}]^2$$



HW-99-12-5-1 (Name) ~~Draw~~ Plot the Given Figure in MATLAB C_D vs. Re_p

Dimensionless Relations.

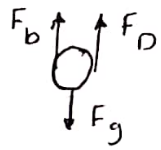
- $Ar = \dfrac{g d_p^3 \rho_f (\rho_p - \rho_f)}{\mu_f^2}$ "Archimedes No." , $Re_p = \dfrac{\rho_f U_t d_p}{\mu_f}$

1 _ creeping flow : $Re_t = \dfrac{Ar}{18}$

2 - Inertial " : $Re_t = \sqrt{3.03\, Ar}$

3 _ All flow Regimes: $Re_t = [-3.809 + (3.809^2 + 1.832\, Ar^{.5})^{.5}]^2$

- At Eq. state, for spherical particle (d_p, ρ_p):



$$\Sigma \vec{F}_p = 0 \Rightarrow F_I + F_g = 0 \Rightarrow F_D + F_b - F_g = 0 \Rightarrow F_D = F_g - F_b$$

$$\left.\begin{array}{l} F_D = \frac{\pi g}{6} d_p^3 (\rho_p - \rho_f) \\ C_D = \dfrac{F_D / \frac{\pi d_p^2}{4}}{\frac{1}{2}\rho_f U_t^2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} C_D = \frac{4}{3}\frac{g d_p}{U_t^2}\cdot\frac{(\rho_p-\rho_f)}{\rho_f} \\ Ar = \cdots \\ Re_t = \cdot \end{array}\right\} \Rightarrow C_D = \frac{4}{3}\cdot\frac{Ar}{Re_t^2} \quad (1)$$

valid for all flow regimes

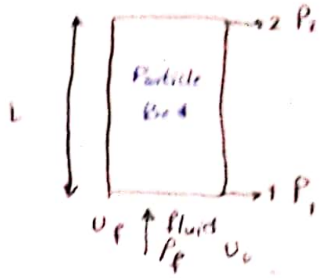
$$Ar = \left[\frac{(Re_t^{.5} + 3.809)^2 - 3.809^2}{1.832}\right]^2 \quad (2)$$

$$(1),(2) \Rightarrow C_D = \frac{4}{3}\cdot\left[\frac{(Re_t^{.5} + 3.809)^2 - 3.809^2}{1.832\, Re_t}\right] \quad (3)$$

HW-99-12-5-2 (Name) Plot C_D (eq.3) vs. Re_t over $Re_t \in [10^{-2} - 10^{10}]$ log-log

Fluid Pressure Loss in Packed Particle Beds

- Fixed bed $\xrightarrow{U_0 \uparrow}$ Fluidized bed



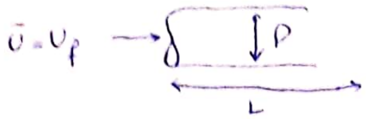
ΔP_{1-2} (measured) $= \Delta P_{fric.} + \rho_f g L \longrightarrow \underbrace{\Delta P_{fric.}}_{\text{the unrecoverable Pressure Loss}} = \Delta P_{1-2} - \rho_f g L$ ①

([illegible]) the unrecoverable Pressure Loss

([illegible] ρ_f [illegible])

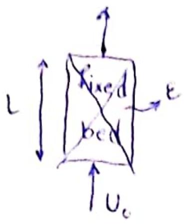
$(\Delta P_{fric} \approx \Delta P_{1-2})$

I) Tube flow analogy : viscous flow conditions ([illegible])



$$\Delta P_{fric.} = \frac{32}{D^2} \mu_f L \cdot \bar{U} \quad ② \quad \text{The Hagen-Poiseuille eq.}$$

$$Q = \bar{U} \cdot \frac{\pi D^2}{4} \quad , \quad \Delta P \propto \bar{U}$$



$$\Delta P = \underbrace{K_p}_{\text{Bed spec.}} \mu_f \cdot L \cdot U_0 \quad ③ \quad \text{Darcy equation} \quad , \quad \Delta P \propto U_0$$

$$\bar{U} \longrightarrow \frac{U_0}{\varepsilon} \quad ④$$

$$D \longrightarrow D_e \quad ⑤ \quad \text{effective diameter}$$

- Permeability $\propto \dfrac{\text{void volume}}{\text{internal surface area}}$

- for a cylinderical tube: $\dfrac{\text{void volume}}{\text{int. surf. area}} = \dfrac{\pi D^2/4 \, L}{\pi D L} = \dfrac{D}{4}$ ⑤

$\Rightarrow$ for other geometries: $D_e = 4 \dfrac{\text{void. volume}}{\text{int. surf. area}}$, for cylinders: $D_e = D$ ⑥

- for a particle bed, d_p, ε, $1\,m^3$ = unit volume . $1 - \varepsilon = N_p \dfrac{\pi d_p^3}{6} \Rightarrow N_p = \dfrac{6(1-\varepsilon)}{\pi d_p^3}$

surf. area for N_p particles $= N_p(\pi d_p^2) = \dfrac{6(1-\varepsilon)}{\pi d_p^3} \times \pi d_p^2 = \dfrac{6(1-\varepsilon)}{d_p}$

$$\Rightarrow D_e = 4 \frac{\varepsilon}{\frac{6(1-\varepsilon)}{\pi d_p^3} \times \pi d_p^2} \Rightarrow D_e = \frac{2 \varepsilon d_p}{3(1-\varepsilon)} \quad ⑦$$

effective diameter for a particle bed

$$\Rightarrow \Delta P_{fric.} = \frac{32}{D_e^2} \mu_f L \left(\frac{U_0}{\varepsilon}\right) ⑧ \Rightarrow \Delta P_{fric.} = \frac{32}{\frac{4 \varepsilon^2 d_p^2}{9(1-\varepsilon)^2}} \mu_f L \left(\frac{U_0}{\varepsilon}\right)$$

$$\Rightarrow \Delta P_{fric.} = 72 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \quad ⑨$$

["viscous flow regime"]

- validated by exp. data $\longrightarrow$ Replace "72" by "150".

$$\Delta P_{fric.} = 150 \frac{\mu_f L \cdot U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \quad ⑩$$

"Blake-Kozeny eq."

II) Tube-flow Analogy: Inertial flow conditions

skin friction ...

$$\Delta P_{fric.} = 4f \frac{L}{D} \cdot \frac{\rho_f \bar{U}^2}{2} \xrightarrow[\bar{U} \to \frac{U_0}{\varepsilon}]{D \to D_e} \Delta P_{fric.} = 4f \frac{L}{\frac{2\varepsilon d_p}{3(1-\varepsilon)}} \frac{\rho_f \cdot \left(\frac{U_0}{\varepsilon}\right)^2}{2} = 3f \frac{\rho_f L U_0^2}{d_p} \frac{(1-\varepsilon)}{\varepsilon^3} \quad (11)$$

• validated by exp. data if $3f = 1.75$

$$\Rightarrow \Delta P_{fric.} = 1.75 \frac{\rho_f L U_0^2}{d_p} \cdot \frac{(1-\varepsilon)}{\varepsilon^3} \quad (12)$$ "Burke-Plummer eq." , $\Delta P \propto U_0^2$

III) Ergun's eq. [1949] (Viscous flow regime + inertial flow regime)

$$\Delta P_{fric.} = \underbrace{150 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3}}_{\text{viscous term}} + \underbrace{1.75 \frac{\rho_f L U_0^2}{d_p} \cdot \frac{(1-\varepsilon)}{\varepsilon^3}}_{\text{inertial term}} \quad (13)$$ 'Ergun's eq.'

• if $Re_p = \frac{\rho_f U_0 d_p}{\mu_f} \Rightarrow \Delta P_{fric} = 1.75 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)}{\varepsilon^3} [85.7(1-\varepsilon) + Re_p] \quad (14)$

• if $\varepsilon \cong .4$
- $Re_p \simeq 50 \Rightarrow 85.7(1-\varepsilon) \approx Re_p \Rightarrow$ viscous forces ~ inertial force
- $Re_p > 50$ inertial forces > viscous forces
- $Re_p < 50$ 〃 〃 < 〃 〃

▨ Fluid Pressure Loss in Expanded Particle Beds ($\Delta P = f(\varepsilon)$)

→ The effect of Tortuosity (τ) ← پیچ و خم

• Ergun's eq. (fixed bed) $\xrightarrow[L, D_e]{\text{Analogy}}$ tube-flow

طول موثر و زمان اقامت بیشتر

• in expanded particle beds: $L_e > L$ (15) ; $\tau = \frac{L_e}{L} \geqslant 1$ (16) → رقیق از لحاظ تعداد ذرات

• difficulties
- $\tau \xrightarrow{\text{how?}} \Delta P \xrightarrow{\text{solution}} L \rightsquigarrow \tau L$
- value τ ??

I) The viscous flow Regime: Revised tube-flow analogy for expanded beds

$D \to D_e$
$U \to U_0/\varepsilon$
$L \to \tau L$

$$\xrightarrow{\text{eq. (9)}} \Delta P_{fric.} = 72 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \tau \quad (17)$$

• τ functionality? $\xrightarrow{\text{solution}}$
- we know: $\tau = \tau(\varepsilon)$
- (Fact): if $\varepsilon \to 1 \Rightarrow L_e \to L$

$\Rightarrow \tau = \frac{1}{\varepsilon}$ (18)

$$\xrightarrow[(18)]{(17)} \Delta P_{fric.} = 72 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^4} \quad (19)$$ (It has overprediction)

$$\xrightarrow[\text{against exp. data}]{\text{validation}} \Delta P_{fric.} = 60 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^4} \quad (20)$$ "Blake-Kozeny eq."

II) Inertial Flow Regime : Revised tube flow analogy for expanded beds :

$$\Delta P_{fric.} = 4f \frac{L}{D} \frac{\rho_f U^2}{2} \quad , \quad f \propto (1-\varepsilon) \Longrightarrow f = c_1 (1-\varepsilon) \quad (21)$$

$$L \longrightarrow \tau L \ , \ D \longrightarrow D_e \ , \ U \longrightarrow U_0/\varepsilon \ , \ \tau \longrightarrow 1/\varepsilon$$

$$\Longrightarrow \Delta P_{fric} = 3c_1 \frac{\rho_f L U_0^2}{d_p} \frac{(1-\varepsilon)^2}{\varepsilon^4} \quad (22)$$

min. fluidization

eq. 22 @ $\varepsilon \cong .4$ $\Longrightarrow$ $3c_1 = 1.17$ (23)

compare with ergun's eq.

$$\Longrightarrow \Delta P_{fric.} = 1.17 \frac{\rho_f L U_0^2}{d_p} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^4} \quad (24)$$

"for expanded beds"

III) Expanded Beds : All flow Regimes (viscous + Inertial)

$$\text{eq. 20 \& 24} \Longrightarrow \Delta P_{fric.} = 60 \frac{\mu_f L U_0}{d_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^4} + 1.17 \frac{\rho_f L U_0^2}{d_p} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^4}$$

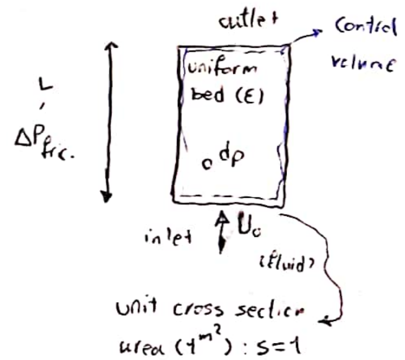
$$\Longrightarrow \underbrace{\Delta P_{fric.}}_{\text{unrecoverable}} = \Big(\underbrace{60 \frac{\mu_f L U_0}{d_p^2}}_{\text{viscous term}} + \underbrace{1.17 \frac{\rho_f L U_0^2}{d_p}}_{\text{inertial term}} \Big) \cdot \Big(\underbrace{\frac{(1-\varepsilon)^2}{\varepsilon^4}}_{\text{voidage function}} \Big) \quad (25)$$

- Note that if $\varepsilon \longrightarrow 1 \Longrightarrow \Delta P_{fric} \longrightarrow 0$?? "this is a paradox"

▨ Relation of Particle Drag Force (f_d) to pressure loss

per particle

- Energy loss ? → Input-output (کلی ← بدون توجه به جزئیات ذرات
  → internal (particle)



I) External view point : Energy loss by the fluid (ΔE)

$$\text{(تعریفی)} \quad \Delta E = \Delta P_{fric.} \times Q_{fluid} \quad (26)$$

$$\Longrightarrow \Delta E_{loss} = (U_0 \times 1) \times \Delta P_{fric.} = U_0 \cdot \Delta P_{fric.} \quad (27)$$

II) Internal view point : ΔE within the control volume

انرژی ازدست رفته از
سیال، هنگام عبور از بستر
به ازای یک ذره

$$\underset{\text{per particle}}{\delta E_p} = f_d \left(\frac{U_0}{\varepsilon}\right) = \text{Irreversible work done by the fluid on a particle} \quad (28)$$

$$N_p = \frac{6(1-\varepsilon)L}{\pi d_p^3} \xrightarrow{(28)} \text{Total energy loss within the bed} = \Delta E_{loss} = N_p \cdot \delta E_p \Longrightarrow \Delta E_{loss} = \frac{6(1-\varepsilon)L}{\pi d_p^3} f_d \cdot \frac{U_0}{\varepsilon} \quad (29)$$

$$\Longrightarrow \Delta E_{loss}\big|_{external} = \Delta E_{loss}\big|_{internal} \xrightarrow{27 \& 29} U_0 \cdot \Delta P_{fric.} = \frac{6(1-\varepsilon)L}{\pi d_p^3} f_d \frac{U_0}{\varepsilon}$$

$$\Longrightarrow f_d \text{ (per particle)} = \frac{\pi d_p^3 \varepsilon}{6(1-\varepsilon)L} \cdot \Delta P_{fric.} \quad (30)$$

" Drag force per particle "

▨ Drag Force

I) Viscous flow conditions:

eq. 20 & 30 $\Rightarrow$ $$f_d = 10\pi d_p \mu_f U_0 \frac{(1-\varepsilon)}{\varepsilon^3} \quad (31)$$ "Drag force per particle in a expanded bed"

$$\Rightarrow f_d = \underbrace{3\pi d_p \mu_f U_0}_{\text{total drag force}} \underbrace{\left[\frac{3.33(1-\varepsilon)}{\varepsilon^3}\right]}_{\text{voidage function}} \quad (32)$$

for isolated (single) particle $\leftarrow$ total drag force

$$\Rightarrow f_d \underset{\text{single particle}}{(\text{unhindered})} = 3\pi d_p \mu_f U_0$$ (laminar, viscous conditions) (33)

$$\Rightarrow f_d (\text{in a bed}) = f_d (\text{unhindered}) \times f(\varepsilon) \quad (34)$$

- if $\varepsilon \to 1 \Rightarrow f_d \to 0$ ✗ ((33) (درحالیکه باید

- if $\varepsilon \cong .4 \Rightarrow f(\varepsilon) \geq 30 \leqslant 30+1$

- Revision of (32) : $$f_d = 3\pi d_p \mu_f U_0 \underbrace{\left[\frac{3.33(1-\varepsilon)}{\varepsilon^3} + 1\right]}_{\substack{\text{New voidage function} \\ \text{(modified)}}} \quad (35)$$ "Modified Drag Force Per Particle"

- now : if $\varepsilon \to 1 \Rightarrow f_d (\text{in a bed}) = f_d (\text{unhindered})$

II) Inertial flow ~~Reg~~ Conditions:

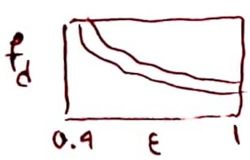
eq. 24 & 30 $\Rightarrow$ $$f_d = 0.055\pi \rho_f d_p^2 U_0^2 \underbrace{\left(\frac{3.55(1-\varepsilon)}{\varepsilon^3}\right)}_{\text{voidage function}} \quad (36)$$

- if $\varepsilon \to 1 \Rightarrow f_d \to 0$ ✗

$$\Rightarrow f_d = 0.055\pi \rho_f d_p^2 U_0^2 \underbrace{\left[\frac{3.55(1-\varepsilon)}{\varepsilon^3} + 1\right]}_{\text{modified voidage function}} \quad (37)$$

$f_d \in [.4 - 1]$

HW-99-12-12-1 (Name) compare the voidage function with the modified voidage function for both flow regimes



$d_p = 160\ \mu m,\ 250\ \mu m,\ 1000\ \mu m$

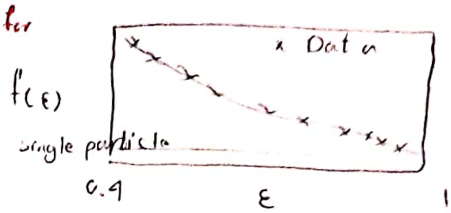
$U_0 = .04\ m/s$

$\mu_f = 1.8 \times 10^{-4}\ g/cm.s$

$\rho_f = .0012\ g/cm^3 \times \frac{kg}{10^3\ g} \times \frac{10^6\ cm^3}{m^3}$

– Converting [modified] voidage functions into ε^{n} :

• HW-99-12-12-2 (Name) curve fit and find n for both regimes ($n = -3.8$)



I) viscous flow regime:

$$\text{eq. } 35 \Longrightarrow \quad f_d = \underbrace{3\pi d_p \mu_f U_o}_{\text{single particle}} \cdot \underbrace{\varepsilon^{-3.8}}_{\text{in a bed}} \qquad (38)$$

II) Inertial flow regime..

$$\text{eq. } 37 \Longrightarrow \quad f_d = .055\pi d_p^2 \rho_f U_o^2 \cdot \varepsilon^{-3.8} \qquad (39)$$

. Note: if $\varepsilon \longrightarrow 1 \Rightarrow$ eq. 38 & 39 : $f_d = f_d$ (single particle)

• C_D for inertial flow regime (when $\varepsilon \longrightarrow 1$) :

$$C_D = \frac{f_d / \pi d_p^2/4}{\rho_f U_o^2/2} = \frac{.055\pi \rho_f d_p^2 U_o^2}{\pi \frac{d_p^2}{4} \rho_f U_o^2/2} \cong .44 \quad ✓$$

– Finding new $\Delta P_{fric.}$ relations;

I) viscous flow regime:

$$\text{eq. } 30 \;\&\; 38 \Rightarrow \quad \Delta P_{fric.} = 18 \frac{\mu_f L U_o}{d_p^2} \cdot (1-\varepsilon) \cdot \varepsilon^{-4.8} \qquad (40)$$

II) inertial flow regime:

$$\text{eq. } 30 \;\&\; 39 \Rightarrow \quad \Delta P_{fric.} = .33 \frac{\rho_f L U_o^2}{d_p} (1-\varepsilon) \cdot \varepsilon^{-4.8} \qquad (41)$$

III) All flow regimes:

$$\Delta P_{fric.} = \Delta P_{fric.}(\text{viscous}) + \Delta P_{fric.}(\text{inertial}) \qquad (42) \qquad \left(Re_p = \frac{\rho_f U_o d_p}{\mu_f}\right)$$

$$\text{eq. } 40 \& 41 \Rightarrow \quad \Delta P_{fric.} = \left(\frac{18}{Re_p} + .33\right) \frac{\rho_f L U_o^2}{d_p} (1-\varepsilon)\varepsilon^{-4.8} \quad \text{"Expanded beds"} \qquad (43)$$

• eq. 43 should be compatible with Ergun's eq.

. HW-99-12-12-3 (Name) compare eq. 43 with Ergun's eq. over ε [.4 – .8]

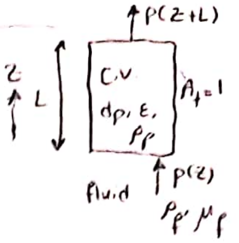
$\rho_p = 1500\ kg/m^3$, $d_p = 150\ \mu m$, 400, 100, $U_o = 4\ cm/s$, 8, 16

$\mu_f \longrightarrow$ air @ $T = 25°c$, ρ_f = Air @ $T = 25°c$, $P = 1\ atm$

'Fil F05 - 51' : 04"

"Homogeneous" ε ثابت در همه‌جا

cross-sectional area



The Steady State balance of forces for a F.B

$\Delta P_{measured} = p(z) - p(z+L)$ ①

at St-St $\Rightarrow \Delta P_{meas} = F \cdot A_t = W_B \cdot A_t$ ②

$W_B = \bar{\rho}_B \cdot (L \times A_t)\, g$ ③ "weight"

$\bar{\rho}_B = \varepsilon \rho_f + (1-\varepsilon)\rho_p$ ④ "mean bulk density"

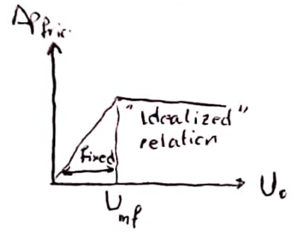
$\Rightarrow W_B = [\varepsilon \rho_f + (1-\varepsilon)\rho_p] L g$ ⑤

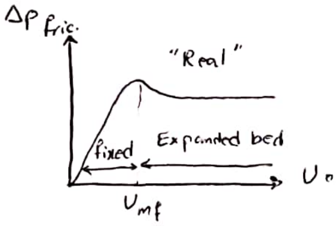
$\Rightarrow \Delta P_{meas.} = [\varepsilon \rho_f + (1-\varepsilon)\rho_p] L g$

$\Delta P_{fric.} = \Delta P_{meas.} - \rho_f \cdot L g$ ⑥ $\Rightarrow \Delta P_{fric.} = [\varepsilon \rho_f + (1-\varepsilon)\rho_p] L g - \rho_f L g$ ⑦

از ترازی نیرو بدست آمد $\Rightarrow \Delta P_{fric.} = (\rho_p - \rho_f)(1-\varepsilon) L g$ ⑧ $\rightarrow V_p = L A_t (1-\varepsilon)$ particles volume in the bed

- over a range of U_0 : $(1-\varepsilon) L = cte$ ⑨ $\Rightarrow \Delta P_{fric.} = cte$ ⑩



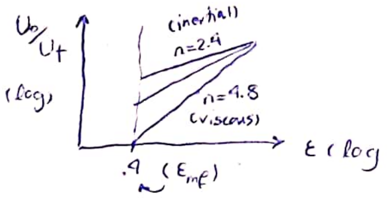


Fixed bed	$\longrightarrow$ Expanded bed	, Min. Fluidization
$V_p = (1-\varepsilon_m) L_m$	$= (1-\varepsilon) L$	$= (1-\varepsilon_{mf}) L_{mf}$
ε_m, L_m	ε, L	ε_{mf}, L_{mf}

Empirical Results

"Richardson - Zaki" : $U_0 = U_t \cdot \varepsilon^n$ ⑫ $\begin{cases} n = 4.8 & \text{(viscous flow regime)} \\ n = 2.4 & \text{(Inertial flow regime)} \end{cases}$ ⑬

. where U_t = Terminal velocity of a single particle (ρ_p, d_p)



$\Rightarrow \dfrac{4.8-n}{n-2.4} = .043\, Ar^{.57}$ ⑭ where $Ar = \dfrac{g d_p^3 \rho_f (\rho_p - \rho_f)}{\mu_f^2}$ ⑮

. at U_0, spec. $\Rightarrow n \Rightarrow \varepsilon$ ✓

. Eq.14 $\begin{cases} \text{Small } Ar \Rightarrow n \rightarrow 4.8 & \text{(viscous flow)} \\ \text{large } Ar \Rightarrow n \rightarrow 2.4 & \text{(inertial flow)} \end{cases}$

I) Viscous flow regime ($\Delta P_{fric.}$ (eq.40) $= \Delta P_{fric}$ (force balance))

$\Delta P_{fric.} = 18 \dfrac{\mu_f L U_0}{d_p^2} (1-\varepsilon) \varepsilon^{-4.8} = (\rho_p - \rho_f)(1-\varepsilon) L g$ ⑰ $\Rightarrow U_0 = \dfrac{(\rho_p - \rho_f) g d_p^2}{18 \mu_f} \cdot \varepsilon^{4.8}$ ⑱

$\Rightarrow U_0 = U_t \cdot \varepsilon^{4.8}$ ⑲ where $U_t = \dfrac{(\rho_p - \rho_f) g d_p^2}{18 \mu_f}$ ⑳ "Terminal velocity of a single particle"

. As $\varepsilon \longrightarrow 1 \Rightarrow U_0 = U_t$ (limit) "single particle in a bed"

II) Inertial flow regime (ΔP_{fric} (eq.41) $= \Delta P_{fric.}$ (force balance))

$\Delta P_{fric.} = .33 \dfrac{\rho_f L U_0^2}{d_p} (1-\varepsilon) \varepsilon^{-4.8} = (\rho_p - \rho_f)(1-\varepsilon) L g$ ㉑ $\Rightarrow U_0 = \sqrt{3.03\, g d_p \left(\dfrac{\rho_p - \rho_f}{\rho_f}\right)} \cdot \varepsilon^{2.4}$ ㉒

$\Rightarrow U_0 = U_t \cdot \varepsilon^{2.4}$ ㉓ where $U_t = \sqrt{3.03\, g d_p \dfrac{(\rho_p - \rho_f)}{\rho_f}}$ for a single particle

HW-99-12-17-1 (Name) Derive a relationship for $U_0 = f(\varepsilon, n)$ for all flow regimes.

III.) U_o as a function of ε for all flow regimes,

$$\Delta P_B|_{frictional} = f(U_o, \varepsilon) \quad (24) \implies \Delta P_B \propto U_o^{a} \varepsilon^{b} \quad , \quad \text{for expanded beds, } (1-\varepsilon)L_B = cte \implies \Delta P_B = cte \quad (25)$$

$$d(\Delta P_B) = \frac{\partial \Delta P_B}{\partial U_o} dU_o + \frac{\partial \Delta P}{\partial \varepsilon} d\varepsilon \quad (26) \xrightarrow[d(\Delta P_B)=0 \ (27)]{\Delta \ (25)} \frac{dU_o}{d\varepsilon} = \frac{-\frac{\partial \Delta P_B}{\partial \varepsilon}}{\frac{\partial \Delta P_B}{\partial U_o}} \quad (28)$$

$$\implies \frac{dU_o}{d\varepsilon} = \frac{-U_o^{a} b \varepsilon^{b-1}}{a U_o^{a-1} \varepsilon^{b}} = \frac{-b}{a} \frac{U_o}{\varepsilon} \quad (29) \quad \text{"O.D.E."}$$

- subject to the following B.C. : $@ \ \varepsilon = 1 \implies U_o = U_T$ (30)
- Thus (29), (30) $\implies U_o = U_T \varepsilon^{-b/a}$ (31) $\xrightarrow{\text{in fact}}$ $U_o = U_T \varepsilon^{n}$ where $n = \frac{-b}{a}$ (32)

The Primary Forces acting on a Fluidized Particle

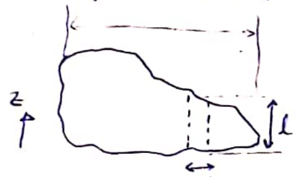
U_T forces $= f(\rho_f, \mu_f, \rho_p, d_p)$

1) Buoyancy Force

$df_b = \Delta p \cdot dA$ (1)

$\frac{dP}{dz}$ in a fluid to be linear function $\implies \frac{dp}{dz} = cte$ (2)

$\implies \Delta p = -\ell \left(\frac{dp}{dz}\right)$ (3)



"Irregular particle"

(1), (3) $\implies df_b = -\ell\left(\frac{dp}{dz}\right)dA$ (4) $\implies f_b = \left(-\frac{dp}{dz}\right)\int_A \ell \, dA = \left(-\frac{dp}{dz}\right)V_p$ (5)

- if $\frac{dp}{dz} \propto \vec{g} \implies \frac{dp}{dz} = -\rho_f g$ (6) , (5), (6) $\implies f_b = V_p \rho_f g$ (7)

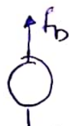
- In a fluidized bed $\rightarrow$ $\frac{dp}{dz} = -\overline{\rho_{bed}} \, g$ (8)
  $\rightarrow$ $f_b = \left(\frac{\pi d_p^3}{6}\right)\overline{\rho_{bed}} \, g$ (9) $= V_p \overline{\rho_{bed}} \, g$

- where: $\overline{\rho}_{bed} = \varepsilon \rho_f + (1-\varepsilon)\rho_p$ (10)

– The effective weight of a fluidized particle:

$$W_{eff.} = \vec{f_g} + \vec{f_b} = -\frac{\pi d_p^3}{6}(\rho_p - \rho_f) g \underset{\sim}{\varepsilon} \quad (12)$$

For a single particle in a column of fluid:
$W_{eff} = -\frac{\pi d_p^3}{6}(\rho_p - \rho_f) g$ (13)



$f_g = \left(\frac{\pi d_p^3}{6}\right) \rho_p \cdot g$ (11) V_p

$\implies W_{eff.} = f(\varepsilon)$

FOG end

2) The Drag Force:

f_d in a fluidized bed $\propto \Delta P_{fric.}$, $\cancel{f_d = 3\pi d_p \mu_f}$

2.1 Viscous flow regime

$$f_d = 3\pi d_p \mu_f U_c \varepsilon^{-3.8} \quad (14) \Rightarrow f_d = 3\pi d_p \mu_f u_t \left(\frac{U_c}{u_t}\right) \cdot \varepsilon^{-3.8} \quad (15)$$ → "single particle hindered"

$$f_d \Big|_{\text{single particle}} = 3\pi d_p \mu_f u_t \quad (16)$$ "single unhindered particle"

$$\text{if } f_d = W_{eff} \Rightarrow f_d = \pi \frac{d_p^3}{6} (\rho_p - \rho_f) g \cdot \varepsilon \quad (17)$$

at limiting: single particle: $$f_d = 3\pi d_p \mu_f u_t = \pi \frac{d_p^3}{6} (\rho_p - \rho_f) g \quad (18)$$

$$\xrightarrow{(15)} f_d = \pi \frac{d_p^3}{6} (\rho_p - \rho_f) g \left(\frac{U_c}{u_t}\right) \varepsilon^{-3.8} \quad (19)$$

2.2 Inertial flow regime

$f_d \equiv$ acting on a particle in a fluidized bed $= .055\, \pi \rho_f d_p^2 U_c^2 \varepsilon^{-3.5}$ (20) $\xrightarrow{\times \frac{U_t^2}{U_t^2}} f_d = .055\pi \rho_f d_p^2 u_t^2 \left(\frac{U_c}{u_t}\right)^2 \varepsilon^{-3.5}$ (21)

at eq.- state $\Rightarrow f_d = W_{eff} = \pi \frac{d_p^3}{6} (\rho_p - \rho_f) g \varepsilon$ (22) "for a single unhindered particle : $\pi \frac{d_p^3}{6} (\rho_p - \rho_f) g$"

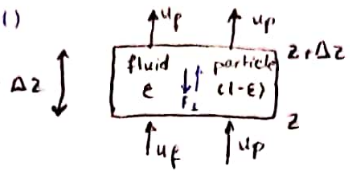
$$(21), (22) \Rightarrow f_d = W_{eff}(\text{single particle}) \cdot \left(\frac{U_c}{u_t}\right)^2 \varepsilon^{-3.5} \quad (23)$$

2.3 All flow regime

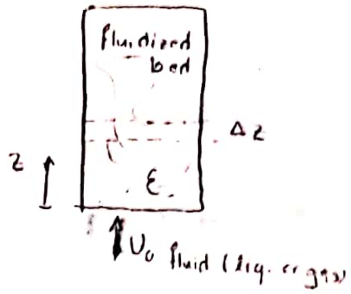
- HW-99-12-19-1 (Name): Derive a relation for f_d (all flow regimes)

A General Formulation

- A ~~…~~ 1D continuum description (Two-fluid model)



cross-sectional area = 1



- Assumptions made
  - → Fluid (continuum phase)
  - → Incomp. fluid
  - → z & t (time) ind variable
  - → Interaction force (F_I). Exchange of linear momentum between two phases per unit volume of bed. $F_I = F_I(u_p, u_f, \varepsilon)$ (1)
  - u_f: Interstitial velocity of fluid phase
  - → u_p: 〃 〃 - particle 〃
  - → unsteady state eqs. of change
  - → 1D eqs. of change
  - → Non-Reactive system

I) Conservation of mass (Continuity eq.)

I.1 Fluid phase

acc = in. - out (2) $\Rightarrow$ $\frac{\partial}{\partial t}(\rho_f \cdot \Delta z \cdot 1 \cdot \varepsilon) = \rho_f u_f \cdot 1 \cdot \varepsilon\big|_z - \rho_f u_f \cdot 1 \cdot \varepsilon\big|_{z+\Delta z}$ (3)

$\xrightarrow[\text{then we get}]{\div \Delta z \text{ then } \lim_{\Delta z \to 0}}$ $\frac{\partial}{\partial t}(\rho_f \varepsilon) = -\frac{\partial}{\partial z}(\rho_f u_f \varepsilon)$ (4)

Incomp. : $\rho_f = cte$ (5) $\Rightarrow$ $\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial z}(u_f \cdot \varepsilon) = 0$ (6) "Fluid phase continuity eq."

I.2 Particle phase

(2) $\Rightarrow$ $\frac{\partial}{\partial t}(\rho_p \, \Delta z \cdot 1 \cdot (1-\varepsilon)) = \rho_p u_p (1 \cdot (1-\varepsilon))\big|_z - \rho_p u_p \, 1 \cdot (1-\varepsilon)\big|_{z+\Delta z}$ (7)

$\xrightarrow[\text{〃 } \rho_p = cte]{\text{〃 〃 〃}}$ $\frac{\partial}{\partial t}(1-\varepsilon) = -\frac{\partial}{\partial z}[u_p(1-\varepsilon)]$ (8)

$\Rightarrow$ $\frac{\partial}{\partial t}(1-\varepsilon) + \frac{\partial}{\partial z}[u_p(1-\varepsilon)] = 0$ (9) "particle phase continuity eq."

overally: $u_f, u_p, \varepsilon = f(t, z)$ (10)

II) Overall Continuity eq.

(6) + (9) $\Rightarrow$ $\cancel{\frac{\partial \varepsilon}{\partial t}} + \frac{\partial}{\partial z}(u_f \cdot \varepsilon) + \cancel{\frac{\partial}{\partial t}(1-\varepsilon)} + \frac{\partial}{\partial z}[u_p(1-\varepsilon)] = 0$ Over all mass balance (O.M.B.)

$\Rightarrow$ $\frac{\partial}{\partial z}[u_f \cdot \varepsilon + u_p(1-\varepsilon)] = 0$ (11) $\Rightarrow$ $u_f \varepsilon + u_p(1-\varepsilon) = \text{constant}$ @ z

at $z = 0$ $\Rightarrow$ constant $= U_0 \times 1$ $\Rightarrow$ $U_0 = u_f \cdot \varepsilon + u_p(1-\varepsilon)$ (12)

$\Rightarrow$ $u_f = \frac{U_0 - u_p(1-\varepsilon)}{\varepsilon} = u_f(z, t)$ (13) : Independent variables ($u_p, u_f, \varepsilon, z, t$)

u_{fp} = slip velocity $= u_f - u_p = \left[\frac{U_0 - u_p(1-\varepsilon)}{\varepsilon}\right] - u_p$ $\Rightarrow$ $u_{fp} = \frac{U_0 - u_p}{\varepsilon}$ (14)

~~F_I~~ F (1) $\Rightarrow$ $F_I = F_I(u_p, \varepsilon)$ (15) "per unit volume* of the bed"

III) Conservation of Linear momentum

– Assumptions made $\longrightarrow$ Neglect Molecular Momentum transfer ($\cancel{\tau}$ or ~~stress~~)
$\quad\quad\quad\quad\quad\quad\quad\;\;\llcorner\!\longrightarrow$ other assumptions made before

$\boxed{\text{III.1}}$ Fluid phase

rate of momentum in – rate of mom. out + Applied forces = rate of acc of mom.

$$\Rightarrow [\rho_f u_f \varepsilon \cdot 1 \cdot u_f]_z - [\rho_f u_f \varepsilon \cdot 1 \cdot u_f]_{z+\Delta z} - F_I(1)\Delta z - \rho_f(\varepsilon \times 1 \times \Delta z) g + p(z)|_z - p(z)|_{z+\Delta z} = \rho_f \frac{\partial}{\partial t}[\rho_f \varepsilon \Delta z u_f] \quad (16)$$

$$\xrightarrow{\div \Delta z \,\&\, \lim_{\Delta z \to 0}} \rho_f \frac{\partial}{\partial t}[u_f \varepsilon] = -\rho_f \frac{\partial}{\partial z}(u_f^2 \varepsilon) - F_I - \rho_f \varepsilon g - \frac{\partial P}{\partial z} \quad (17)$$

$$\Rightarrow \rho_f \frac{\partial}{\partial t}(u_f \varepsilon) + \rho_f \frac{\partial}{\partial z}(\varepsilon u_f^2) = -\frac{\partial P}{\partial z} - F_I - \rho_f \varepsilon g \quad (18)$$

$\llcorner$ per unit volume

$\boxed{\text{III.2}}$ Particle phase

$$[\rho_p u_p (1-\varepsilon) u_p \times 1]_z - [\rho_p u_p (1-\varepsilon) u_p \times 1]_{z+\Delta z} + F_I(\Delta z \times 1) - \rho_p(1-\varepsilon)\Delta z g = \frac{\partial}{\partial t}[\rho_p u_p (1-\varepsilon)\Delta z] \quad (19)$$

$$\xrightarrow{\div \Delta z \,\&\, \lim_{\Delta z \to 0}} \rho_p \frac{\partial}{\partial t}[u_p(1-\varepsilon)] + \rho_p \frac{\partial}{\partial z}[u_p^2(1-\varepsilon)] = F_I - (1-\varepsilon)\rho_p g \quad (20)$$

IV) Unknowns $\{u_f, u_p, \varepsilon, P\}$ as a function of (z,t) , $F_I = F_I(u_p, \varepsilon)$ (21)

– Governing eqs. $\longrightarrow$ mass balances : Eqs. 6 & 9
$\quad\quad\quad\quad\quad\;\;\llcorner\!\longrightarrow$ momentum " : " . 18 & 20

– Initial Conditions : @ $\forall z$, $t=0$: $u_p(0)$, $u_f(0)$, $\varepsilon(0)$, $p(0)$

– Boundary " : if $\overset{\checkmark}{u_p}$ & $\overset{\checkmark}{\varepsilon} \Rightarrow \overset{\checkmark}{F_I}(u_p, \varepsilon) \longrightarrow$ { Particle phase: Eqs. 9 & 20, P×, $F_I = F_d$ } $\Rightarrow \overset{\checkmark}{u_p}, \overset{\checkmark}{\varepsilon} \Longrightarrow$ finally $\Rightarrow \overset{\checkmark}{u_f}, \overset{\checkmark}{P}$

Eqs. 12 & 18

Eq. 12, $\overset{\checkmark}{u_f} \Rightarrow$ Eq. 18, $\overset{\checkmark}{P}$

$\boxed{\text{IV.1}}$ Particle phase

– Mass balance $\xrightarrow{\text{Eq. 9}}$

$$-\frac{\partial \varepsilon}{\partial t} - u_p \frac{\partial \varepsilon}{\partial z} + (1-\varepsilon)\frac{\partial u_p}{\partial z} = 0$$

$$\frac{\partial \varepsilon}{\partial t} + u_p \frac{\partial \varepsilon}{\partial z} - (1-\varepsilon)\frac{\partial u_p}{\partial z} = 0 \quad (22)$$

$\{\varepsilon, u_p\}$ = unknowns

– Momentum balance $\xrightarrow{\text{Eq. 20}}$

$$\rho_p\left\{u_p \frac{\partial}{\partial t}(1-\varepsilon) + (1-\varepsilon)\frac{\partial u_p}{\partial t}\right\} + \rho_p\left\{u_p^2 \frac{\partial}{\partial z}(1-\varepsilon) + (1-\varepsilon)\frac{\partial u_p^2}{\partial z}\right\} = \underbrace{F_I - (1-\varepsilon)\rho_p g}_{\equiv F} \quad (23)$$

$$\xrightarrow{(22),(23)} (1-\varepsilon)\rho_p\left[\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z}\right] = F \quad (24)$$

• where $F \triangleq F_I - (1-\varepsilon)\rho_p g$ (25) $\xrightarrow{\text{became}}$ $F_I = F_I(u_p, \varepsilon) \Rightarrow F = F(u_p, \varepsilon)$ (26)

✗ If Eqs. 22 & 24 subject to I.C.s & B.C.s $\Longrightarrow \overset{\checkmark}{u_p}, \overset{\checkmark}{\varepsilon}$ (@ z,t)

Then using Eqs. 12 & 18 $\Rightarrow \overset{\checkmark}{u_f}$ & $\overset{\checkmark}{P}$

Stability of Fluidized Beds

- $U_o = U_{mf} \xrightarrow{U_o \uparrow} U_o > U_{mf}$, Homogeneous F.B. (Stable, ε) $\xrightarrow[\text{where?}]{U_o \uparrow}$ Void Volume (Bubble) / Bubbling Regime (Unstable)

(1) Linear Stability Analysis for systems of DEs

- Such a system is of the generic form: $\dot{X}(t) = f(t, X)$, $t \geq 0$, $X(0) = X_0$

  such that : $X = [x_1(t), x_2(t), \dots, x_n(t)]^T$, $f = [f_1, f_2, \dots, f_n]^T$ (1) (Generally Nonlinear function)

  $X_0 \equiv$ A specified I.C. for the system

- In General, the stability analysis of the system depends greatly on the form of $f(t, X)$
- In a special case: $f(\cancel{t}, X) \rightarrow f(X)$ "Autonomous system" Then Eq. 1 becomes:

  $\dot{X}(t) = f(X)$, $t \geq 0$, $X(0) = X_0$ (2) , $f(X)$ (Generally Nonlinear function of X)

- At Equilibrium point (C or X_e) we get: $\dot{X}\big|_{eq.} = 0$ or $f(X_e) = f(C) = 0$ (3)

  The system may be linearized about "C" by using Taylor's expansion theorem:

  $\therefore$ $f(X) = f(C) + Df(C).(X - C) + R(\bar{X})$ (4) (by neglecting the higher order terms of (4), it becomes:)

  $\Rightarrow f(X) \cong f(C) + Df(C).(X - C)$ (5) where

$$Df(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{n \times n} \quad (6)$$

= A matrix of 1^{st} order partial derivations of $f(x)$

(3), (5) $\Rightarrow$ $f(x) \cong Df(C).(X - C)$ (7)

(2), (7) $\Rightarrow$ $\dot{X}(t) \cong Df(C).(X - C)$ (8)

let us $X - C \triangleq \tilde{X}$ (9) , $\dot{X} = \dot{\tilde{X}}$ $\Rightarrow$ $\dot{\tilde{X}} = Df(C)\tilde{X}$ $\Rightarrow$ $\dot{X} = AX$ (9) (linear system)

Stability ? $\det(Df(C) - \lambda I) = 0 \Rightarrow \lambda_1, \lambda_2, \dots, \lambda_n$ "eigen values $= \lambda_i = \underbrace{\alpha_i}_{\text{Real}} + \underbrace{\beta_i i}_{\text{Imaginary}}$"

If $Re(\overset{\text{all}}{\forall} \lambda_i) < 0$ then the linearized system is stable

If $Re(\underset{\text{some}}{\exists} \lambda_i) > 0$ " " " " " unstable

(2) Stability analysis of Fluidized Beds

- For particle phase:
  - continuity eq. : $\frac{\partial \varepsilon}{\partial t} + u_p \frac{\partial \varepsilon}{\partial z} - (1-\varepsilon)\frac{\partial u_p}{\partial z} = 0$ (1)
  - momentum eq. : $(1-\varepsilon)\rho_p[\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z}] = F$ (2)

  } Governing equations

  - where: $F \equiv F_I - (1-\varepsilon)\rho_p g$ (3) "Nonlinear function"

- Equilibrium point $\begin{cases} \varepsilon = \varepsilon_o = cte \\ u_p = 0 \end{cases}$
  $\Rightarrow$ Eq. 1 : $0 + 0 - 0 = 0$ ✓
  $\Rightarrow$ Eq. 2 : $0 + 0 = F_{eq}$ ✓ $\Rightarrow$ $F_{eq.} = 0$ (4)
  $\Rightarrow$ $F_I^{eq} - (1-\varepsilon_o)\rho_p g$ $\Rightarrow$ $F_I^{eq.} = (1-\varepsilon_o)\rho_p g$ (5) @ st-st (eq. point)

overall continuity eq. $\Rightarrow$ $U_o = \varepsilon u_f + (1-\varepsilon)u_p \xrightarrow[\text{point}]{\text{eq.}} U_o = \varepsilon_o u_f \Rightarrow u_f\big|_{eq.} = \frac{U_o}{\varepsilon_o}$ (6)

if @ $U_o = U_{mf}$, $u_p = 0$ $\Rightarrow$ $u_{f,mf} = \frac{U_{mf}}{\varepsilon_{mf}}$ (7)

$\left| \begin{array}{l} \varepsilon_o \longrightarrow \varepsilon_o + \varepsilon^* \\ u_p \longrightarrow 0 + u_p^* \end{array} \right.$ (8) $\quad , \|\varepsilon^*\| \ll \varepsilon_o$ $\quad \|u_p^*\|$ small $\Longrightarrow \left| \begin{array}{l} \hat{\varepsilon} = \varepsilon_o + \varepsilon^* \\ \hat{u}_p = 0 + u_p^* \end{array} \right.$ (9) "small perturbation" اغتشاش

(1) $\Rightarrow \dfrac{\partial \hat{\varepsilon}}{\partial t} + u_p \dfrac{\partial \hat{\varepsilon}}{\partial z} - (1-\hat{\varepsilon}) \dfrac{\partial \hat{u}_p}{\partial z} = 0$ (10) 'perturbed continuity eq. for particle phase'

$\Rightarrow \dfrac{\partial \varepsilon^*}{\partial t} + u_p^* \dfrac{\partial \varepsilon^*}{\partial z} - (1-\varepsilon_o-\varepsilon^*) \dfrac{\partial u_p^*}{\partial z} = 0 \Rightarrow \dfrac{\partial \varepsilon^*}{\partial t} + \underbrace{u_p^* \dfrac{\partial \varepsilon^*}{\partial z}}_{\text{small}} - (1-\varepsilon_o)\dfrac{\partial u_p^*}{\partial z} + \underbrace{\varepsilon^* \dfrac{\partial u_p^*}{\partial z}}_{\text{small}} = 0$ (11)

$\Rightarrow \dfrac{\partial \varepsilon^*}{\partial t} = (1-\varepsilon_o) \dfrac{\partial u_p^*}{\partial z}$ (12) "Disturbed continuity eq. for particle phase"

(2) $\Rightarrow (1-\hat{\varepsilon})\rho_p \left[\dfrac{\partial \hat{u}_p}{\partial t} + \hat{u}_p \dfrac{\partial \hat{u}_p}{\partial z} \right] = F(\hat{u}_p, \hat{\varepsilon})$ (13) $\Rightarrow (1-\varepsilon_o-\varepsilon^*)\rho_p \left[\dfrac{\partial u_p^*}{\partial t} + \underbrace{u_p^* \dfrac{\partial u_p^*}{\partial z}}_{\text{small}} \right] = \underbrace{F(\hat{u}_p, \hat{\varepsilon})}_{\text{Nonlinear function}}$ (14)

$\Rightarrow (1-\varepsilon_o)\rho_p \dfrac{\partial u_p^*}{\partial t} = F(\hat{u}_p, \hat{\varepsilon}) = F(0+u_p^*, \varepsilon_o+\varepsilon^*)$ (15) "Disturbed momentum eq. for particle phase"

using Taylor's expansion theorem $\Rightarrow F(\hat{u}_p, \hat{\varepsilon}) = F(0+u_p^*, \varepsilon_o+\varepsilon^*)\Big|_{eq.} + \dfrac{\partial F}{\partial u_p}\Big|_{eq.}(\hat{u}_p - 0) + \dfrac{\partial F}{\partial \varepsilon}\Big|_{eq.}(\hat{\varepsilon} - \overset{\varepsilon_o}{\cancel{0}}) + R$

$\Rightarrow F(\hat{u}_p, \hat{\varepsilon}) \cong F\big|_{eq} + f_{up} \cdot u_p^* + f_\varepsilon \cdot (\hat{\varepsilon} - \varepsilon_o)$ (16)

(4) $\Rightarrow \left| \begin{array}{l} F(\hat{u}_p, \hat{\varepsilon}) \cong 0 + u_p^* \cdot f_{up} + \varepsilon^* \cdot f_\varepsilon \quad (17) \\ \text{where: } f_{up} = \dfrac{\partial F}{\partial u_p}\Big|_{\substack{\varepsilon=\varepsilon_o \\ u_p=0 \\ eq.}} \ \& \ f_\varepsilon = \dfrac{\partial F}{\partial \varepsilon}\Big|_{\substack{\varepsilon=\varepsilon_o \\ u_p=0 \\ eq.}} \end{array} \right.$ "known" (18)

(15), (17) $\Rightarrow (1-\varepsilon_o)\rho_p \dfrac{\partial u_p^*}{\partial t} \cong u_p^* \cdot f_{up} + \varepsilon^* \cdot f_\varepsilon$ (19) "Approximation of Disturbed momentum eq."

f_{up} & f_ε sign

- f_{up}
  - @ eq. point: $u_p = 0$, a small increase in u_p
  - $u_p \uparrow$ @ $u_p = 0$ $\longrightarrow$ $u_{s1} = u_f - 0$ @ eq. point
    - $u_p \uparrow$ $u_{s2} = u_f - u_p$ (small)
    - $u_{s2} < u_{s1}$ $\Longrightarrow$ $f_{up} < 0$
- f_ε
  - @ eq. point: $u_p = 0$, $\varepsilon = \varepsilon_o$
  - if $\varepsilon = \varepsilon_o + \varepsilon^*$ $\Longrightarrow$ $f_\varepsilon < 0$

(20)

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for particle phase

- $\dfrac{\partial \varepsilon^*}{\partial t} = (1-\varepsilon_o) \dfrac{\partial u_p^*}{\partial z}$ (21) "perturbed continuity eq."
- $(1-\varepsilon_o) \dfrac{\partial u_p^*}{\partial t} = \dfrac{u_p^*}{\rho_p} f_{up} + \dfrac{\varepsilon^*}{\rho_p} f_\varepsilon$ (22) " ~ momentum eq."

$\dfrac{\partial (21)}{\partial t}$, $\dfrac{\partial (22)}{\partial z}$ $\Rightarrow \left| \begin{array}{l} \dfrac{\partial^2 \varepsilon^*}{\partial t^2} = (1-\varepsilon_o) \dfrac{\partial^2 u_p^*}{\partial t \partial z} \quad (23) \\ (1-\varepsilon_o) \dfrac{\partial^2 u^*}{\partial t \partial z} = \dfrac{f_{up}}{\rho_p} \cdot \dfrac{\partial u_p^*}{\partial z} + \dfrac{f_\varepsilon}{\rho_p} \cdot \dfrac{\partial \varepsilon^*}{\partial z} \quad (24) \end{array} \right\} \Rightarrow \dfrac{\partial^2 \varepsilon^*}{\partial t^2} = \dfrac{f_{up}}{\rho_p} \cdot \dfrac{\partial u_p^*}{\partial z} + \dfrac{f_\varepsilon}{\rho_p} \cdot \dfrac{\partial \varepsilon^*}{\partial z}$ (25)

(21), (25) $\Longrightarrow \dfrac{\partial^2 \varepsilon^*}{\partial t^2} = \dfrac{f_{up}}{\rho_p} \left(\dfrac{1}{1-\varepsilon_o} \cdot \dfrac{\partial \varepsilon^*}{\partial t} \right) + \dfrac{f_\varepsilon}{\rho_p} \cdot \dfrac{\partial \varepsilon^*}{\partial z}$ (26) "single parameter" $\Longrightarrow \dfrac{\partial^2 \varepsilon^*}{\partial t^2} - \underbrace{\left(\dfrac{f_{up}}{\rho_p} \times \dfrac{1}{(1-\varepsilon_o)} \right)}_{\equiv B} \dfrac{\partial \varepsilon^*}{\partial t} - \underbrace{\dfrac{f_\varepsilon}{\rho_p}}_{\equiv C} \cdot \dfrac{\partial \varepsilon^*}{\partial z} = 0$ (27)

– then Eq. 27 becomes: $\dfrac{\partial^2 \varepsilon^*}{\partial t^2} + B \dfrac{\partial \varepsilon^*}{\partial t} + C \dfrac{\partial \varepsilon^*}{\partial z} = 0$ (28) "perturbed, combined, continuity + momentum eqs."

where: $\varepsilon^* = \varepsilon^*(t, z)$ in the bed

because f_{up} & $f_\varepsilon < 0 \Rightarrow B$ & $C > 0$ "always" (29)

- let us $\varepsilon^* = \varepsilon_A \exp[at + ik(z - vt)]$ (30) "input function" / where $\varepsilon^* = \varepsilon^*(z,t)$ (31) / $i = \sqrt{-1}$

ε_A = initial wave amplitude / a = amplitude growth rate / k = wave No. $= \dfrac{2\pi}{\lambda} > 0$

λ = wave length / v = wave velocity > 0

$$\Rightarrow \varepsilon^* = \varepsilon_A \cdot \exp[(a - ikv)t] \cdot \exp(ikz) \quad (32)$$

$$\left|\begin{array}{l} \dfrac{\partial \varepsilon^*}{\partial z} = ik.\varepsilon^* \\ \dfrac{\partial \varepsilon^*}{\partial t} = (a - ikv).\varepsilon^* \\ \dfrac{\partial^2 \varepsilon^*}{\partial t^2} = (a - ikv)^2 \cdot \varepsilon^* \end{array}\right. \quad (33)$$

(28), (33) $\Rightarrow (a - ikv)^2 \varepsilon^* + B(a - ikv)\varepsilon^* + Ck\varepsilon^* i = 0$ (34)

$\Rightarrow [a^2 - k^2v^2 + Ba] + i[-2akv - Bkv + Ck] = 0$ (35) "$\alpha + i\beta$"

$$\Rightarrow \left| \begin{array}{l} \text{Real part} = 0 \\ \text{Img.} \quad " \quad = 0 \end{array}\right. \Rightarrow \left| \begin{array}{ll} a^2 - k^2v^2 + Ba = 0 & (36) \\ -2akv - Bkv + Ck = 0 & (37) \end{array}\right. \Rightarrow \left| \begin{array}{l} a = \dfrac{C - Bv}{2v} \\ k^2 = \dfrac{C^2 - B^2v^2}{4v^4} \end{array}\right. \quad (38)$$

$$k^2 > 0 \Rightarrow \frac{C^2 - B^2v^2}{4v^4} > 0 \Rightarrow C^2 - B^2v^2 > 0 \Rightarrow C > Bv \quad (39) \xRightarrow{(38)} a > 0 \quad (40)$$

(40) $\Rightarrow$ Hom. F. B $\Rightarrow$ Destabalized F.B. // Final Decision

The Primary Force Interactions

(A) Under equilibrium conditions:

1 Drag Force per particle: $f_d = \frac{\pi d_p^3}{6}(\rho_p - \rho_f)\left(\frac{U_0}{u_t}\right)^{\frac{4.8}{n}} \cdot \underbrace{\varepsilon^{-3.8}}_{\text{curve fitting term}}$ (1)

$n = \begin{cases} 4.8 & \text{for viscous flow} \\ 2.4 & \text{for inertial flow} \end{cases}$

2 Net effective weight:

$w_{eff} = f_g + f_b$ (2) $\Longrightarrow w_{eff} = \cancel{f} - \frac{\pi d_p^3}{6}(\rho_p - \rho_f) g \cdot \varepsilon$ (3)

3 Total Force per particle (Eqs. 1, 3): $f_o = f_d + w_{eff}$

$\Longrightarrow f_o = \frac{\pi d_p^3}{6}(\rho_p - \rho_f)\left(\frac{U_0}{u_t}\right)^{\frac{4.8}{n}} \cdot \varepsilon^{-3.8} - \frac{\pi d_p^3}{6}(\rho_p - \rho_f) g \cdot \varepsilon$ (4)

+ At st-st conditions: $f_o = 0$ $\Longrightarrow \left(\frac{U_0}{u_t}\right)^{\frac{4.8}{n}} \varepsilon^{-3.8} = \varepsilon$ (5) $\Longrightarrow U_0 = u_t \cdot \varepsilon^n$ (6)

for all flow regimes

"St-st expansion law"

(B) Non-equilibrium conditions:

1 Drag force (u_{fp}) : $u_{fp} = u_f - u_p$ (7) , Total cont. eq.: $U_0 = \varepsilon u_f + (1-\varepsilon) u_p$ (8)

Slip velocity of a particle in a F.B.

(7), (8) $\Longrightarrow u_{fp} = \frac{U_0 - (1-\varepsilon) u_p}{\varepsilon} - u_p \Longrightarrow u_{fp} = \frac{U_0 - u_p}{\varepsilon}$ (9)

$\frac{U_0}{\varepsilon} \longrightarrow u_{fp}$

or $\varepsilon u_{fp} = u_0 - u_p$

2 $f_b = -\overset{\text{particle volume}}{V_p} \frac{dp}{dz}$ (10)

$\xRightarrow{(3), (8)} f_o = \frac{\pi d_p^3}{6}\left[(\rho_p - \rho_f) g \left(\frac{U_0 - u_p}{u_t}\right)^{\frac{4.8}{n}} \varepsilon^{-3.8}\right] - \frac{\pi d_p^3}{6} \cdot \frac{\partial p}{\partial z} - \rho_p g \frac{\pi d_p^3}{6}$

$\Longrightarrow f_o \text{ (per particle)} = \frac{\pi d_p^3}{6}\left[(\rho_p - \rho_f) g \left(\frac{U_0 - u_p}{u_t}\right)^{\frac{4.8}{n}} \cdot \varepsilon^{-3.8} - \frac{\partial p}{\partial z} - \rho_p g\right]$ (11)

3 No. of particle per unit volume of F.B. (for ε, d_p, ρ_p). $N_p = \frac{6(1-\varepsilon)}{\pi d_p^3}$ (12)

$\Longrightarrow$ Total Force per unit volume of F.B.: $F_o = f_o \times N_p$ (13)

$\Longrightarrow F_o = (1-\varepsilon)\left[(\rho_p - \rho_f) g \left(\frac{U_0 - u_p}{u_t}\right)^{\frac{4.8}{n}} \cdot \varepsilon^{-3.8} - \frac{\partial p}{\partial z} - \rho_p g\right]$ (14)

Fluid-Dynamic Elasticity of the Particle Phase

"Elastic response" $\varepsilon \rightsquigarrow 1 - \varepsilon \equiv \alpha$ (15) فضای پُر

The penetration Distance (Δz)

(1D: z-only)

$\underset{\text{effective}}{\alpha_e} \triangleq \alpha - \left(\frac{\partial \alpha}{\partial z}\right) \Delta z$ (16)

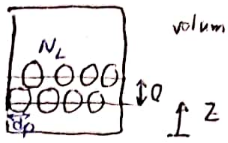
• under Eq. conditions, we get: $\alpha_e = \alpha$ (17) i.e.: $\frac{\partial \alpha}{\partial z} = 0$

Modified Interaction Force between the fluid and solid phases

$$f(\alpha_e) = f(\alpha - \frac{\partial \alpha}{\partial z} \cdot \Delta z) \approx f(\alpha) - \frac{\partial f}{\partial \alpha}\left(\frac{\partial \alpha}{\partial z} \Delta z\right) \quad (18)$$

"linear approximation of $f(\alpha_e)$"

Δz ?



volume $\equiv \theta \times 1$

↳ unit cross section $(s=1)$

– An estimation for Δz

$$\alpha_{vol.} = \frac{N_L \times \frac{\pi d_p^3}{6}}{\theta \times 1} \quad (19) \qquad \alpha_{surf.} = \frac{N_L \times \frac{\pi d_p^2}{4}}{1} \quad (20)$$

If suppose that $\alpha_{vol.} = \alpha_{surf.} \Longrightarrow \theta = \frac{2}{3} d_p$ (21)

• As an assumption $\Delta z = \theta = \frac{2}{3} d_p$ (22)

– Eqs (18), (22) $\Rightarrow f^+ \equiv f(\alpha_e) = f - \frac{2}{3} d_p \left(\frac{\partial f}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial z}\right)$ (23) "per particle"

If $F^+ \equiv$ per unit volume of bed $= N_p \times f^+ = \frac{6\alpha}{\pi d_p^3}\left[f - \frac{2}{3} d_p \left(\frac{\partial f}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial z}\right)\right]$ (24)

$$\Rightarrow F^+ = \frac{6\alpha}{\pi d_p^3} f - \frac{6\alpha}{\pi d_p^3}\left[\frac{2}{3} d_p \left(\frac{\partial f}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial z}\right)\right] \quad (25)$$

where $\alpha = 1 - \varepsilon$ (26)

we had: $f = \frac{\pi d_p^3}{6}\left[(\rho_p - \rho_f) g \left(\frac{U_0 - u_p}{u_t}\right)^{\frac{4.8}{n}} \cdot \varepsilon^{-3.8} - \frac{\partial P}{\partial z} - \rho_p g\right]$ (27)

(25) $\Rightarrow F^+ = F - E \frac{\partial \alpha}{\partial z}$ (28)

Elasticity Modulus $= E \triangleq \frac{4\alpha}{\pi d_p^2} \times \frac{\partial f}{\partial \alpha}$ (29)

$\triangleq \frac{-4(1-\varepsilon)}{\pi d_p^2} \times \frac{\partial f}{\partial \varepsilon}$

• by definition: $\frac{\partial P_p}{\partial z} \triangleq E \frac{\partial \alpha}{\partial z}$ (30)

$\triangleq -E \frac{\partial \varepsilon}{\partial z}$

• If (equilibrium), $f_0 = 0$, then: $U_0 = u_t \cdot \varepsilon^n$ "st-st expansion law"

H.W.: Derive: $E = 3.2 g d_p (1-\varepsilon)(\rho_p - \rho_f)$

H.W. 1400-1-17-1 (Name)

The Dynamic Wave Velocity (u_D) for particle phase in terms of E.

• By definition: $E \triangleq \rho_p u_D^2$ (32)

$$\Rightarrow u_D \triangleq \sqrt{\frac{E}{\rho_p}} = \sqrt{3.2 g d_p (1-\varepsilon) \frac{(\rho_p - \rho_f)}{\rho_p}} \quad (33)$$

$u_D \equiv$ "Dynamic Wave Velocity"

• If the fluid to be a gas phase, then $u_D = \sqrt{3.2 g d_p (1-\varepsilon)}$ (34), because $\rho_p \gg \rho_f$

– Additional term to be considered in the momentum eqs.: $\rho_p u_D^2 \frac{\partial \varepsilon}{\partial z}$

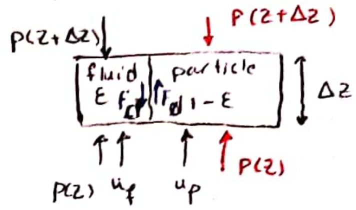
(28) $\Rightarrow F^+ = F + E \frac{\partial \varepsilon}{\partial z}$ (35)

$$= F + \underbrace{\rho_p u_D^2 \frac{\partial \varepsilon}{\partial z}}$$

Additional term should be considered in the momentum balance

Modified Particle Bed Model

- Note: The assumptions made are the same as before: "No stress"



(I) Fluid Phase:

- Continuity eq. : $\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial z}(\varepsilon u_f) = 0$ (1)
- Momentum eq. : $\varepsilon \rho_f \left[\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z}\right] = -F_d - \varepsilon \frac{\partial P}{\partial z} - \varepsilon \rho_f g$ (2)

unknowns $\begin{cases} \varepsilon \\ u_f \\ u_p \\ P \end{cases}$

O.M.B. : $U_0 = \varepsilon u_f + (1-\varepsilon) u_p$ (5)

(II) Particle phase:

- continuity eq. : $-\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial z}\left[(1-\varepsilon) u_p\right] = 0$ (3)
- momentum eq.: $(1-\varepsilon)\rho_p \left[\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z}\right] = +F_d \underbrace{-(1-\varepsilon)\frac{\partial P}{\partial z}}_{\text{New}} - (1-\varepsilon)\rho_p g \underbrace{+\rho_p u_D^2 \frac{\partial \varepsilon}{\partial z}}_{\text{New}}$ (4)

A simplified procedure : For Gas-Solid F.B.s

- we have $\rho_p \gg \rho_f$ ($\le \rho_g$)
- thus eq. (2) $\Rightarrow$ $\varepsilon \rho_f [LHS] \ll RHS \Longrightarrow 0 \cong -F_d - \varepsilon \frac{\partial P}{\partial z} - 0 \Rightarrow F_d \cong -\varepsilon \frac{\partial P}{\partial z}$ (6)

  In this case we have: $-\frac{\partial P}{\partial z} \simeq F_d/\varepsilon$ (7)

- Eqs. (4), (7) $\Rightarrow$ $(1-\varepsilon)\rho_p\left[\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z}\right] = +F_d + (1-\varepsilon)\frac{F_d}{\varepsilon} - (1-\varepsilon)\rho_p g + \rho_p u_D^2 \frac{\partial \varepsilon}{\partial z}$

  $\Rightarrow LHS = \frac{F_d}{\varepsilon} - (1-\varepsilon)\rho_p g + \rho_p u_D^2 \frac{\partial \varepsilon}{\partial z}$ (8)

- let us define: $F \triangleq \frac{F_d}{\varepsilon} - (1-\varepsilon)\rho_p g$ (9), then eq. (8) becomes:

$$\begin{cases} LHS = F + \rho_p u_D^2 \frac{\partial \varepsilon}{\partial z} & (10) \\ -\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial z}\left[(1-\varepsilon)u_p\right] = 0 & (11) \end{cases} \Longrightarrow \begin{cases} u_p,\ \varepsilon \\ 2\text{ eqs.} \end{cases}$$

- at eq. point : $\begin{cases} \varepsilon = \varepsilon_0 = cte \\ u_p = 0 \end{cases}$ (12) $\Longrightarrow \begin{cases} (11): & 0 + 0 = 0 \checkmark \\ (10): & 0 = F_{eq.} + 0 \Rightarrow F_{eq.} = 0 \Rightarrow F_d^{eq.} = \underset{\varepsilon_0}{\varepsilon}(1-\underset{\varepsilon_0}{\varepsilon})\rho_p g \end{cases}$ (13)

- $F_d = f_d \times N_p = (1-\varepsilon)\left[(\rho_p - \rho_f) g \left(\frac{U_0 - \overset{0}{u_p}}{u_t}\right)^{\frac{4.8}{n}} \cdot \varepsilon^{-3.8}\right]$ (14)

(13), (14) $\Rightarrow$ $U_0 = u_t \cdot \varepsilon^n$ "st-st. expansion law"

Stability Analysis

. Disturbances $\begin{cases} \varepsilon = \varepsilon_o + \varepsilon^* \\ u_p = 0 + u_p^* \end{cases}$ (15) where both $\{\varepsilon^* \& u_p^*\}$ are small values relative to $(\varepsilon_o \& 0)$

- cont. eq. particle phase : $-\frac{\partial}{\partial t}(\varepsilon_o+\varepsilon^*)+(1-\varepsilon_o-\varepsilon^*)\frac{\partial}{\partial z}(0+u_p^*)+(0+u_p^*)\frac{\partial}{\partial z}(1-\varepsilon_o-\varepsilon^*)=0$ (16)

- Momentum eq. " " : $(1-\varepsilon_o-\varepsilon^*)\rho_p\left[\frac{\partial}{\partial t}(0+u_p^*)+(0+u_p^*)\frac{\partial}{\partial z}(0+u_p^*)\right]=F(u_p^*,\varepsilon^*)+\rho_p u_D^2\frac{\partial}{\partial z}(\varepsilon_o+\varepsilon^*)$ (17)

where $F=F(u_p^*,\varepsilon^*)$ "Nonlinear functionality"

(16) $\Rightarrow -\frac{\partial\varepsilon^*}{\partial t}+(1-\varepsilon_o)\frac{\partial u_p^*}{\partial z}-\underbrace{\varepsilon^*\frac{\partial u_p^*}{\partial z}}_{\approx 0}+\underbrace{u_p^*\frac{\partial}{\partial z}(-\varepsilon^*)}_{\approx 0}=0$ (18)

(18) $\Rightarrow \frac{\partial\varepsilon^*}{\partial t}=(1-\varepsilon_o)\frac{\partial u_p^*}{\partial z}$ (19) "perturbed continuity eq. for particle phase"

(17) $\Rightarrow (1-\varepsilon_o)\rho_p\left[\frac{\partial u_p^*}{\partial t}+\underbrace{u_p^*\frac{\partial u_p^*}{\partial z}}_{\approx 0}\right]-\underbrace{\varepsilon^*\rho_p\left[\frac{\partial u_p^*}{\partial t}+u_p^*\frac{\partial u_p^*}{\partial z}\right]}_{\approx 0}=F(u_p^*,\varepsilon^*)+\rho_p u_p^2\frac{\partial\varepsilon^*}{\partial z}$ (20)

(20) $\Rightarrow (1-\varepsilon_o)\rho_p\frac{\partial u_p^*}{\partial t}\cong \underbrace{F(u_p^*,\varepsilon^*)}_{\text{Nonlinear}}+\rho_p u_D^2\frac{\partial\varepsilon^*}{\partial z}$ (21) "perturbed momentum eq. for particle phase

where $F\Big|_{\text{eq. point}}^{(\varepsilon=\varepsilon_o,\,u_p=0)}=0$ (22)

$$F(u_p^*,\varepsilon^*)=\underbrace{F\Big|_{\text{eq. point}}^{\varepsilon=\varepsilon_o,\,u_p=0}}_{=0}+\overbrace{\frac{\partial F}{\partial u_p}\Big|_{\text{eq. point}}}^{\text{value}}(u_p^*)+\overbrace{\frac{\partial F}{\partial\varepsilon}\Big|_{\text{eq. point}}}^{\text{value}}(\varepsilon^*)+\text{H.O.T.} \quad (23)$$

(23) $\Rightarrow F\cong u_p^* f_{u_p}+\varepsilon^* f_\varepsilon$ (24)

(21),(24) $\Rightarrow (1-\varepsilon_o)\frac{\partial u_p^*}{\partial t}\cong\frac{u_p^*}{\rho_p}f_{u_p}+\frac{\varepsilon^*}{\rho_p}f_\varepsilon+u_D^2\frac{\partial\varepsilon^*}{\partial z}$ (25) where

$f_{u_p}=\frac{\partial F}{\partial u_p}\Big|_{\text{eq. point}}^{\varepsilon=\varepsilon_o,\,u_p=0}\equiv cte$ (26)

$f_\varepsilon=\frac{\partial F}{\partial\varepsilon}\Big|_{\text{eq. point}}^{\varepsilon=\varepsilon_o,\,u_p=0}\equiv cte$ (27)

. In addition we had: $F\triangleq\frac{F_d}{\varepsilon}-(1-\varepsilon)\rho_p g$ (28)

where: $F_d=(1-\varepsilon)\left[\underbrace{(\rho_p-\rho_f)}_{\text{for gas }\cong\rho_p}g\left(\frac{U_o-u_p}{u_t}\right)^{\frac{4.8}{n}}\varepsilon^{-3.8}\right]$ (29)

(26) $\Rightarrow f_{u_p}=\frac{\partial F}{\partial u_p}\Big|_{\text{eq. point}}^{\varepsilon=\varepsilon_o,\,u_p=0}=\frac{\partial F}{\partial u_p}\Big|_{U_o=u_t\varepsilon_o^n}$ (30)

(30) $\Rightarrow f_{u_p}=\frac{\partial}{\partial u_p}\left[F_d/\varepsilon-(1-\varepsilon)\rho_p g\right]_{eq.}=\frac{\partial}{\partial u_p}\left[F_d/\varepsilon\right]_{\varepsilon=\varepsilon_o}^{U_o=u_t\varepsilon_o^n}$ (31)

- HW-00-1-22-1) Derive: $f_{u_p}=\frac{-4.8(1-\varepsilon_o)\rho_p g}{\varepsilon_o\, n.u_t}\varepsilon_o^{1-n}<0$ (32)

$(27) \Rightarrow f_\varepsilon = \frac{\partial F}{\partial \varepsilon}\Big|_{eq.\ point}^{u_t = u_t \varepsilon_c} = \frac{\partial}{\partial \varepsilon}\left[\frac{F_d}{\varepsilon} - (1-\varepsilon)\rho_p g\right]_{eq.}$ (33)

- H.W-00-1-22-2) Derive $f_\varepsilon = -4.8\rho_p g \frac{1-\varepsilon_0}{\varepsilon_0} < 0$ (34)

$(32).(34) \Rightarrow f_{up} = \frac{\varepsilon_0 f_\varepsilon}{\varepsilon_0 n u_t}\varepsilon_0^{1-n} = f_\varepsilon \frac{\varepsilon_0^{1-n}}{n u_t}$ (35) , $f_{up} = \frac{f_\varepsilon}{n u_t \varepsilon_0^{n-1}} \cdot \frac{1-\varepsilon_0}{1-\varepsilon_0}$ (36)

• Let us define: $u_k \triangleq n u_t \varepsilon_0^{n-1}(1-\varepsilon_0) > 0$ (37) "Kinetic Wave Velocity" $\Big\} \Rightarrow f_{up} = \frac{f_\varepsilon(1-\varepsilon_0)}{u_k}$ (38)

$(24).\ F \cong f_{up} u_p^* + f_\varepsilon \varepsilon^*$ (39) $\xrightarrow{(32),(34)} F \cong \frac{-4.8\rho_p g(1-\varepsilon_0)}{\varepsilon_0 n u_t}\varepsilon_0^{1-n} u_p^* - 4.8\rho_p g \frac{1-\varepsilon_0}{\varepsilon_0}\varepsilon^*$ (40)

$\Rightarrow F \cong -\frac{4.8 g(1-\varepsilon_0)\rho_p}{u_k \cdot \varepsilon_0}\left[u_k \varepsilon^* + (1-\varepsilon_0)u_p^*\right]$ (41)

• Let: $D \triangleq \frac{4.8 g(1-\varepsilon_0)}{u_k \varepsilon_0} > 0$ (42) $\Rightarrow F \cong -D\rho_p\left[u_k \varepsilon^* + (1-\varepsilon_0)u_p^*\right]$ (43)

- Perturbed governing eqs
  - ✓ cont. eq. for particle phase: $\frac{\partial \varepsilon^*}{\partial t} = (1-\varepsilon_0)\frac{\partial u_p^*}{\partial z}$ (44)
  - ✓ mom. eq. " " " : $\rho_p(1-\varepsilon_0)\frac{\partial u_p^*}{\partial t} = -D\rho_p\left[u_k \varepsilon^* + (1-\varepsilon_0)u_p^*\right] + \rho_p u_p^2 \frac{\partial \varepsilon^*}{\partial z}$ (45)

$\frac{\partial (44)}{\partial t} \Rightarrow \frac{\partial^2 \varepsilon^*}{\partial t^2} = (1-\varepsilon_0)\frac{\partial^2 u_p^*}{\partial t . \partial z}$ (46)

$\frac{\partial (45)}{\partial z} \Rightarrow (1-\varepsilon_0)\frac{\partial^2 u_p^*}{\partial z . \partial t} = -D\left[u_k \frac{\partial \varepsilon^*}{\partial z} + \frac{\partial \varepsilon^*}{\partial t}\right] + u_D^2 \frac{\partial^2 \varepsilon^*}{\partial z^2}$ (47) (44)

$(46),(47) \xrightarrow{u_p^* \text{ حذف}} \frac{\partial^2 \varepsilon^*}{\partial t^2} = -D\left[u_k \frac{\partial \varepsilon^*}{\partial z} + \frac{\partial \varepsilon^*}{\partial t}\right] + u_D^2 \frac{\partial^2 \varepsilon^*}{\partial z^2}$ (48)

$\Rightarrow \frac{\partial^2 \varepsilon^*}{\partial t^2} - u_D^2 \frac{\partial^2 \varepsilon^*}{\partial z^2} + D\left[\frac{\partial \varepsilon^*}{\partial t} + u_k \frac{\partial \varepsilon^*}{\partial z}\right] = 0$ (49) • Combined and Perturbed Momentum equation for particle phase"

- $\varepsilon^* = \varepsilon_A \cdot \exp[(a - ikv)t + ikz]$ (50)
  - ε_A: initial wave amplitude
  - a: amplitude growth rate
  - v: wave velocity
  - k: wave No. $= \frac{2\pi}{\lambda} > 0$, λ: wave length

$\Rightarrow$
$$\frac{\partial \varepsilon^*}{\partial z} = ik\varepsilon^*$$
$$\frac{\partial \varepsilon^*}{\partial t} = (a - ikv)\varepsilon^*$$
$$\frac{\partial^2 \varepsilon^*}{\partial z^2} = ik(ik\varepsilon^*) = -k^2\varepsilon^*$$
$$\frac{\partial^2 \varepsilon^*}{\partial t^2} = (a - ikv)^2\varepsilon^*$$ (51)

$(49),(51) \Rightarrow [a^2 - k^2v^2 + u_D^2k^2 + aD] + i[-2akv - Dkv + Du_k k] = 0$ (52) $\alpha + i\beta = 0$

$\beta = 0 \Rightarrow a = \frac{D}{2v}(u_k - v)$ (53)

$\alpha = 0 \Rightarrow k^2 = \frac{a^2 + Da}{v^2 - u_D^2}$ (54) $\Rightarrow k^2 = \frac{D^2}{4v^2}\left[\frac{u_k^2 - v^2}{v^2 - u_D^2}\right]$ (55)

I) for short waves: λ small $\Rightarrow k \to \infty$
II) for long waves: λ large $\Rightarrow k \to 0$ $\Big\}$ (56) $k = \frac{2\pi}{\lambda}$

for $k \to \infty$, then $(55) \Rightarrow v^2 - u_D^2 \cong 0 \Rightarrow v = u_D$ (57)

for $k \to 0$, then $(55) \Rightarrow u_k^2 - v^2 \cong 0 \Rightarrow u_k = v$ (58)

Stability Criterion

(53): $a = \frac{D}{2V}(u_k - v)$ (59)

- for a stable F.B., we have $a<0$. then $v > u_k$ (60) $\Rightarrow v^2 > u_k^2 \Rightarrow u_k^2 - v^2 < 0$ (61)

for a stable F.B., we should have $k^2 > 0 \Rightarrow v^2 - u_D^2 < 0 \Rightarrow v^2 < u_D^2 \Rightarrow v < u_D$ (62)

- Stability Condition → (60), (62) $\Rightarrow u_k < v < u_D$

→ then the F.B. is @ stable conditions: $u_D > u_k$ (63)

- For an Unstable F.B. → we have: $a > 0$

→ $\Rightarrow u_k > v \Rightarrow u_k^2 > v^2 \Rightarrow u_k^2 - v^2 > 0$ (64)

→ Because $k^2 > 0 \Rightarrow v^2 - u_D^2 > 0 \Rightarrow v > u_D$ (65)

→ $u_k > u_D$ (66)

(63), (66) $\Rightarrow$ Incipient bubbling flow regime: $u_k = u_D$ (67)

Summary of F.B. Stability Determination

1. Input system parameter values: $\rho_p, d_p, \rho_f, \mu_f$
2. To Evaluate parameter "n": $Ar = g d_p^3 \rho_g \frac{(\rho_p - \rho_g)}{\mu_f^2} \longrightarrow u_t = [-3.809 + \sqrt{3.809^2 + 1.832 Ar^{.5}}]^2 \frac{\mu_g}{\rho_g d_p}$

   $n = \frac{4.8 + .1032 Ar^{.57}}{1 + .043 Ar^{.57}}$
3. Set ε @ ε_{mf}: $\varepsilon \approx .4$
4. Evaluate u_k & u_D: $u_D = \sqrt{\frac{E}{\rho_p}} = \sqrt{3.2 g d_p (1-\varepsilon) \frac{\rho_p - \rho_g}{\rho_p}}$, $u_k = n u_t (1-\varepsilon)\varepsilon^{n-1}$
5. Conclusions
   → If $u_k > u_D$: The bed starts to bubble at ε_{mb}
   → If $u_k < u_D$: The bed is initially Hom. (stable)
6. To find ε_{mb}, progressively increase ε, repeating step 4 until $u_k = u_D$: $\varepsilon = \varepsilon_{mb}$, $U_o = U_{mb}$

- HW-00-1-22-3)

particle phase → $d_p = [250, 400, 700, 1000, 2000]$ μm
→ $\rho_p = [1500, 2000, 2500, 3000]$ kg/m^3

Fluid phase → Air, μ_g, ρ_g @ handbooks
→ $T = [25, 50, 75, 100, 200]$ °c
→ $P = [1, 5, 7, 10, 30, 60, 100]$ atm

✓ find ε_{mb}, U_{mb}
✓ effect of d_p
✓ effect of ρ_p
✓ effect of T
✓ effect of P

* Interpretations should be conducted

- Previous Analysis, we supposed that: $\begin{cases} \rho_p \gg \rho_f \\ \text{Incompressible fluid} \end{cases}$

. so we get: $U_o = \varepsilon U_f + (1-\varepsilon)u_p$ (1) Total continuity eq. , $U_f = \dfrac{U_o-(1-\varepsilon)U_p}{\varepsilon}$ (2)

. If $\rho_p \gg \rho_f$, we get: $F_D \cong -\varepsilon \dfrac{\partial P}{\partial z}$ (3)

$\rho_p \gg \rho_f$ X

- The governing eqs. for both the phases could be given by:

I) Particle Phase:

1. Continuity eq. : $-\dfrac{\partial \varepsilon}{\partial t} + \dfrac{\partial}{\partial z}[(1-\varepsilon)u_p] = 0$ (4)

2. Momentum eq. : $(1-\varepsilon)\rho_p\left[\dfrac{\partial u_p}{\partial t} + u_p\dfrac{\partial u_p}{\partial z}\right] = F_D - (1-\varepsilon)\dfrac{\partial P}{\partial z} - (1-\varepsilon)\rho_p g + \rho_p u_D^2 \dfrac{\partial \varepsilon}{\partial z}$ (5)

coupled P.D.E.s

II) Fluid Phase:

1. Continuity eq. : $\dfrac{\partial \varepsilon}{\partial t} + \dfrac{\partial}{\partial z}(\varepsilon u_f) = 0$ (6)

2. Momentum eq. : $\varepsilon\rho_f\left[\dfrac{\partial u_f}{\partial t} + u_f\dfrac{\partial u_f}{\partial z}\right] = -F_D - \varepsilon\dfrac{\partial P}{\partial z} - \varepsilon\rho_f g$ (7)

• unknowns $\{u_f, u_p, \varepsilon, P\}$.

(5), (7) $\Rightarrow \begin{cases} \rho_p[\quad] = \dfrac{F_D}{1-\varepsilon} - \dfrac{\partial P}{\partial z} - \rho_p g + \dfrac{\rho_p u_D^2}{1-\varepsilon}\cdot\dfrac{\partial \varepsilon}{\partial z} & (8) \\ \rho_f[\quad] = \dfrac{-F_D}{\varepsilon} - \dfrac{\partial P}{\partial z} - \rho_f g & (9) \end{cases}$

(8) - (9) $\Rightarrow \rho_p\left[\dfrac{\partial u_p}{\partial t} + u_p\dfrac{\partial u_p}{\partial z}\right] - \rho_f\left[\dfrac{\partial u_f}{\partial t} + u_f\dfrac{\partial u_f}{\partial z}\right] = \dfrac{F_D}{1-\varepsilon} + \dfrac{F_D}{\varepsilon} - \rho_p g + \rho_f g + \dfrac{\rho_p u_D^2}{1-\varepsilon}\cdot\dfrac{\partial \varepsilon}{\partial z}$ (10)

$\Rightarrow \begin{cases} \text{LHS} = \dfrac{F_D}{\varepsilon(1-\varepsilon)} - g(\rho_p - \rho_f) + \dfrac{\rho_p u_D^2}{1-\varepsilon}\cdot\dfrac{\partial \varepsilon}{\partial z} & (11) \\ (4) , (6) \qquad \text{unknowns } \{u_f, u_p, \varepsilon\} & (12) \end{cases}$ " Combined Momentum eq. "

Stability Analysis

- Equilibrium point $\begin{cases} \varepsilon = \varepsilon_o = cte \\ u_p = 0 \\ u_f = u_f = cte \checkmark \Rightarrow u_{f_o} = U_o/\varepsilon_o \end{cases}$ (13)

(11) $\xrightarrow{@ \text{ eq.}}$ $0 + 0 - 0 = \dfrac{F_D^{eq.}}{\varepsilon_o(1-\varepsilon_o)} - g(\rho_p - \rho_f) + 0$ (14) $\Longrightarrow$ $F_D^{eq.} = g\varepsilon_o(1-\varepsilon_o)(\rho_p - \rho_f)$ (15)

if $\rho_f \ll \rho_p \Rightarrow F_D^{eq.} = g\varepsilon_o(1-\varepsilon_o)\rho_p$ (16) $\Longrightarrow$ $F_D^{eq.} = f_D \cdot N_p \Rightarrow U_o = u_t\varepsilon^n$ (17)

(4) $\xrightarrow{@ \text{ eq.}}$ $0 + 0 = 0$ ✓

(6) $\xrightarrow{@ \text{ eq.}}$ $0 + 0 = 0$ ✓

$$\begin{cases} \varepsilon = \varepsilon_o + \varepsilon^* \\ u_p = 0 + u_p^* \\ u_f = u_{f_o} + u_f^* \end{cases} \quad (18)$$ where $\|\varepsilon^*\|$, $\|u_p^*\|$, and $\|u_f^*\|$ are so small values

$$(4) \Rightarrow -\frac{\partial}{\partial t}(\varepsilon_o + \varepsilon^*) + \frac{\partial}{\partial z}[(1-\varepsilon_o-\varepsilon^*)(0+u_p^*)] = 0 \quad (19)$$

$$\Rightarrow \frac{\partial \varepsilon^*}{\partial t} = (1-\varepsilon_o)\frac{\partial u_p^*}{\partial z} \quad (20) \quad \text{or} \quad \frac{\partial u_p^*}{\partial z} = \frac{1}{1-\varepsilon_o}\cdot\frac{\partial \varepsilon^*}{\partial t} \quad (21)$$ Perturbed cont. eq. for particle phase

$$(6) \Rightarrow \frac{\partial}{\partial t}(\varepsilon_o + \varepsilon^*) + \frac{\partial}{\partial z}[(\varepsilon_o+\varepsilon^*)(u_{f_o}+u_f^*)] = 0 \quad (22) \quad \Rightarrow \quad \frac{\partial \varepsilon^*}{\partial t} + \varepsilon_o\frac{\partial u_f^*}{\partial z} + u_{f_o}\frac{\partial \varepsilon^*}{\partial z} = 0 \quad (23)$$ Perturbed cont. eq. for fluid phase

$$(11): \ \rho_p\left[\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial z}\right] - \rho_f\left[\frac{\partial u_f}{\partial t} + u_f\frac{\partial u_f}{\partial z}\right] = F + \frac{\rho_p u_D^2}{1-\varepsilon}\cdot\frac{\partial \varepsilon}{\partial z} \quad (24)$$ where $F \triangleq \frac{F_D}{\varepsilon_o(1-\varepsilon_o)} - g(\rho_p - \rho_f)$ (25)

$$(24) \xRightarrow{@\text{ eq.}} \ 0 - 0 = F^{eq.} + 0 \Rightarrow F^{eq.} = 0 \quad (26)$$, and $F = F(\varepsilon, u_p)$ (27)

$$F_D = (1-\varepsilon)\left[(\rho_p-\rho_f)g\left(\frac{U_o - u_p}{u_t}\right)^{\frac{4.8}{n}}\varepsilon^{-3.8}\right] \quad (28)$$ "Nonlinear" for all regimes

$$(24) \Rightarrow \rho_p\left[\frac{\partial}{\partial t}(0+u_p^*) + (0+u_p^*)\cdot\frac{\partial}{\partial z}(0+u_p^*)\right] - \rho_f\left[\frac{\partial}{\partial t}(u_{f_o}+u_f^*) + (u_{f_o}+u_f^*)\cdot\frac{\partial}{\partial z}(u_{f_o}+u_f^*)\right]$$

$$= F(0+u_p^*, \varepsilon_o+\varepsilon^*) + \frac{\rho_p u_D^2}{1-\varepsilon_o-\varepsilon^*}\cdot\frac{\partial}{\partial z}(\varepsilon_o+\varepsilon^*) \quad (29)$$

$$\Rightarrow \rho_p\left[\frac{\partial u_p^*}{\partial t} + 0\right] - \rho_f\left[\frac{\partial u_f^*}{\partial t} + u_{f_o}\frac{\partial u_f^*}{\partial z}\right] = \underline{F(u_p^*, \varepsilon_o+\varepsilon^*)} + \frac{\rho_p u_D^2}{1-\varepsilon_o}\frac{\partial \varepsilon^*}{\partial z} \quad (30)$$

$$F(0+u_p^*, \varepsilon_o+\varepsilon^*) \approx \underbrace{F(0,\varepsilon_o)\big|_{eq.}}_{=0} + \frac{\partial F}{\partial u_p}\bigg|_{F^{eq.}=0}^{eq.}\cdot u_p^* + \frac{\partial F}{\partial \varepsilon}\bigg|_{F^{eq.}=0}^{eq.}\cdot\varepsilon^* + \text{H.O.T.} \quad (31)$$

$$\Rightarrow F(u_p^*, \varepsilon_o+\varepsilon^*) \cong u_p^* f_{u_p} + \varepsilon^* f_\varepsilon \quad (32)$$

$$(30),(32) \Rightarrow \rho_p\frac{\partial u_p^*}{\partial t} - \rho_f\left[\frac{\partial u_f^*}{\partial t} + u_{f_o}\frac{\partial u_f^*}{\partial z}\right] \cong u_p^* f_{u_p} + \varepsilon^* f_\varepsilon + \frac{\rho_p u_D^2}{1-\varepsilon_o}\frac{\partial \varepsilon^*}{\partial z} \quad (33)$$

? ?

$$f_{u_p} \triangleq \frac{\partial F}{\partial u_p}\bigg|_{F^{eq.}=0}^{eq.} = \frac{\partial}{\partial u_p}\left[\frac{F_D}{\varepsilon(1-\varepsilon)} - g(\rho_p-\rho_f)\right]_{F^{eq.}=0}^{eq.} = \frac{\partial}{\partial u_p}\left[\frac{F_D}{\varepsilon(1-\varepsilon)}\right]^{eq.}$$

$$= \frac{\partial}{\partial u_p}\left[(\rho_p-\rho_f)g\left(\frac{U_o-u_p}{u_t}\right)^{\frac{4.8}{n}}\cdot\varepsilon^{-4.8}\right]^{eq.}, \quad eq. = \begin{cases}\varepsilon=\varepsilon_o \\ u_f = u_{f_o} \\ u_p = 0\end{cases}$$

$$\Rightarrow f_{u_p} = \frac{-4.8g(1-\varepsilon_o)(\rho_p-\rho_f)\varepsilon_o^{-1}}{n u_t (1-\varepsilon_o)\varepsilon_o^{n-1}} = \frac{-4.8g(1-\varepsilon_o)(\rho_p-\rho_f)\varepsilon_o^{-1}}{u_k} < 0 \quad (35)$$

$$n u_t (1-\varepsilon_o)\varepsilon_o^{n-1} \triangleq u_k$$

$$f_\varepsilon \triangleq \frac{\partial F}{\partial \varepsilon}\bigg|_{F^{eq.}=0}^{eq.} = \frac{\partial}{\partial \varepsilon}\left[\frac{F_D}{\varepsilon(1-\varepsilon)} - g(\rho_p-\rho_f)\right]^{eq.} \quad (36) \quad = \frac{\partial}{\partial \varepsilon}\left[\frac{F_D}{\varepsilon(1-\varepsilon)}\right]^{eq.}$$

$$\Rightarrow f_\varepsilon = -4.8(\rho_p-\rho_f)g\,\varepsilon_o^{-1} < 0 \quad (37)$$

$$(35),(37) \Rightarrow f_{u_p} = \frac{f_\varepsilon(1-\varepsilon_o)}{u_k} \quad (38)$$

(32) $\Rightarrow F(u_p^*, \varepsilon_0+\varepsilon^*) \simeq -\dfrac{4.8g(\rho_p-\rho_f)}{u_k\varepsilon_0}\left[u_p^*(1-\varepsilon_0)+u_k\varepsilon^*\right]$ (39)

(33), (39) $\Rightarrow \rho_p\dfrac{\partial u_p^*}{\partial t}-\rho_f\left[\dfrac{\partial u_f^*}{\partial t}+u_{f_0}\dfrac{\partial u_f^*}{\partial z}\right] \simeq -\dfrac{4.8g(\rho_p-\rho_f)}{u_k\varepsilon_0}\left[u_p^*(1-\varepsilon_0)+u_k\varepsilon^*\right]+\dfrac{\rho_p u_D^2}{1-\varepsilon_0}\cdot\dfrac{\partial\varepsilon^*}{\partial z}$ (40)

$\dfrac{\partial}{\partial t}$ (21) $\Rightarrow \dfrac{\partial^2 u_p^*}{\partial t\partial z}=\dfrac{1}{1-\varepsilon_0}\dfrac{\partial^2\varepsilon^*}{\partial t^2}$ (41)

$\dfrac{\partial}{\partial t}$ (23) $\Rightarrow \dfrac{\partial^2 u_f^*}{\partial t\partial z}=\dfrac{-1}{\varepsilon_0}\cdot\dfrac{\partial^2\varepsilon^*}{\partial t^2}-\dfrac{u_{f_0}}{\varepsilon_0}\cdot\dfrac{\partial^2\varepsilon^*}{\partial t\partial z}$ (42)

Total cont. eq. : $U_0=\varepsilon u_f+(1-\varepsilon)u_p$ (43) $\xrightarrow{\text{pert.}}$ $cte=(\varepsilon_0+\varepsilon^*)(u_{f_0}+u_f^*)+(1-\varepsilon_0-\varepsilon^*)(0+u_p^*)$ (44)

$\dfrac{\partial}{\partial t}$ (44) $\Rightarrow \dfrac{\partial u_f^*}{\partial t}=-\dfrac{1-\varepsilon_0}{\varepsilon_0}\cdot\dfrac{\partial u_p^*}{\partial t}-\dfrac{u_{f_0}}{\varepsilon_0}\dfrac{\partial\varepsilon^*}{\partial t}$ (45)

$\dfrac{\partial}{\partial z}$ (40) $\Rightarrow \rho_p\dfrac{\partial^2 u_p^*}{\partial t\partial z}-\rho_f\left[\dfrac{\partial^2 u_f^*}{\partial t\partial z}+u_{f_0}\dfrac{\partial^2 u_f^*}{\partial z^2}\right]=-\dfrac{4.8g(\rho_p-\rho_f)}{\varepsilon_0 u_k}\left[(1-\varepsilon_0)\dfrac{\partial u_p^*}{\partial z}+u_k\dfrac{\partial\varepsilon^*}{\partial z}\right]+\dfrac{\rho_p u_D^2}{1-\varepsilon_0}\cdot\dfrac{\partial^2\varepsilon^*}{\partial z^2}$ (46)

(46), (41), (42), (45) $\Rightarrow \rho_p\left[\dfrac{1}{1-\varepsilon_0}\dfrac{\partial^2\varepsilon^*}{\partial t^2}\right]-\rho_f\left[\dfrac{-1}{\varepsilon_0}\cdot\dfrac{\partial^2\varepsilon^*}{\partial t^2}-\dfrac{u_{f_0}}{\varepsilon_0}\dfrac{\partial^2\varepsilon^*}{\partial t\partial z}+u_{f_0}\cdot\dfrac{\partial}{\partial z}\left(\dfrac{-1}{\varepsilon_0}\dfrac{\partial\varepsilon^*}{\partial t}-\dfrac{u_{f_0}}{\varepsilon_0}\dfrac{\partial\varepsilon^*}{\partial z}\right)\right]$

$=-\dfrac{4.8g(\rho_p-\rho_f)}{\varepsilon_0 u_k}\left[(1-\varepsilon_0)\left(\dfrac{1}{1-\varepsilon_0}\cdot\dfrac{\partial\varepsilon^*}{\partial t}\right)+u_k\dfrac{\partial\varepsilon^*}{\partial z}\right]+\dfrac{\rho_p u_D^2}{1-\varepsilon_0}\cdot\dfrac{\partial^2\varepsilon^*}{\partial z^2}$ (47)

Finally we get: $\dfrac{\partial^2\varepsilon^*}{\partial t^2}+2V\dfrac{\partial^2\varepsilon^*}{\partial t\partial z}+G\dfrac{\partial^2\varepsilon^*}{\partial z^2}+D\left(\dfrac{\partial\varepsilon^*}{\partial t}+u_k\dfrac{\partial\varepsilon^*}{\partial z}\right)=0$ (48)

where
- $V\triangleq\dfrac{\rho_f u_{f_0}(1-\varepsilon_0)}{\varepsilon_0\rho_p+(1-\varepsilon_0)\rho_f}>0$ "weighted mean velocity"
- @ eq. : $u_{f_0}=U_0/\varepsilon_0$ or @ mf: $u_{f_0}=\dfrac{U_{mf}}{\varepsilon_{mf}}$
- $G\triangleq\dfrac{\rho_f u_{f_0}(1-\varepsilon_0)-\rho_p u_D^2\varepsilon_0}{\varepsilon_0\rho_p+(1-\varepsilon_0)\rho_f}$ <0 or >0
- $D\triangleq\dfrac{4.8g(\rho_p-\rho_f)(1-\varepsilon_0)}{u_k(\varepsilon_0\rho_p+(1-\varepsilon_0)\rho_f)}>0$

(49)

\- $\varepsilon^*=\varepsilon_A\cdot\exp[(a-ikv)t+ikz]$ (50)

$$\begin{cases}\dfrac{\partial\varepsilon^*}{\partial t}=(a-ikv)\varepsilon^*\\[2mm]\dfrac{\partial^2\varepsilon^*}{\partial t^2}=(a-ikv)^2\varepsilon^*\\[2mm]\dfrac{\partial^2\varepsilon^*}{\partial t\partial z}=(a-ikv)ik\varepsilon^*\\[2mm]\dfrac{\partial\varepsilon^*}{\partial z}=ik\varepsilon^*\\[2mm]\dfrac{\partial^2\varepsilon^*}{\partial z^2}=-k^2\varepsilon^*\end{cases}\quad(51)$$

(98) , (51) $\Rightarrow$ $(a-ikv)^2\varepsilon^* + 2V'[(a-ikv)ik]\varepsilon^* + G(-k^2\varepsilon^*) + D[(a-ikv)\varepsilon^* + u_k ik\varepsilon^*] = 0$ (52)

$\Rightarrow [a^2 - k^2v^2 + 2Vk^2v - k^2G + Da] + i[-2akv + 2Vak - Dkv + Du_k k] = 0$ (53) $\quad \alpha + i\beta = 0 \Rightarrow \begin{cases}\alpha = 0\\ \beta = 0\end{cases}$

$\alpha = 0 \Rightarrow k^2 = \dfrac{a^2 + Da}{v^2 - 2vV + G}$ (54)

$\beta = 0 \Rightarrow a = \dfrac{Dkv - Du_k k}{-2kv + 2kV} = \dfrac{D(u_k - v)}{2(v-V)} > 0$ or < 0 (55)

$$\Rightarrow k^2 = \frac{\dfrac{D^2(u_k-v)^2}{4(v-V)^2} + \dfrac{D^2(u_k-v)}{2(v-V)}}{v^2 - 2vV + G}$$

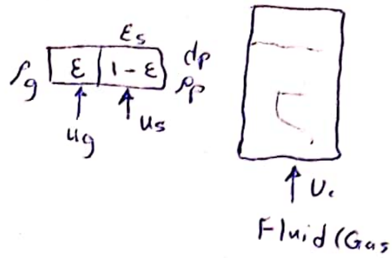
HW-00-1-24-1) Derive: $\quad k^2 = \dfrac{D^2}{4(v-V)^2}\left[\dfrac{(u_k-V)^2 - (v-V)^2}{(v-V)^2 - (V^2-G)}\right]$ (56)

HW-00-1-24-2) Propose a procedure for ε_{mb} or U_{mb}

If $V^2 - G \triangleq (u_{DT} - V)^2$ & $\begin{cases}\hat{v} \triangleq v - V\\ \hat{u}_k \triangleq u_k - V\\ \hat{u}_{DT} \triangleq u_{DT} - V\end{cases}$ Relative velocity with respect to V

HW-00-1-24-3) Derive $\begin{cases} a = \dfrac{D}{2\hat{v}}(\hat{u}_k - \hat{v}) \\ k^2 = \dfrac{D^2}{4\hat{v}^2} \cdot \dfrac{\hat{u}_k^2 - \hat{v}^2}{\hat{v}^2 - \hat{u}_{DT}^2}\end{cases}$

-- Modeling of Gas-Solid F.B. using TFM approach [3D]



Governing Eq.s:

I) Continuity eq.:

1) Gas Phase: $$\frac{\partial}{\partial t}(\varepsilon_g \rho_g) + \vec{\nabla}\cdot(\varepsilon_g \rho_g \vec{u_g}) = 0 \qquad (1)$$

2) solid " : $$\frac{\partial}{\partial t}(\varepsilon_s \rho_s) + \vec{\nabla}\cdot(\varepsilon_s \rho_s \vec{u_s}) = 0 \qquad (2)$$

, where . $\varepsilon_s + \varepsilon_g = 1$ (3)

. Total continuity eq.: $$\frac{\partial}{\partial t}(\varepsilon_s \rho_s + \varepsilon_g \rho_g) + \vec{\nabla}\cdot(\varepsilon_g \rho_g \vec{u_g} + \varepsilon_s \rho_s \vec{u_s}) = 0 \qquad (3\text{-}a)$$

II) Momentum eqs. considering the viscous forces (continuum App.)

1) Gas phase: $$\frac{\partial}{\partial t}(\varepsilon_g \rho_g \vec{u_g}) + \vec{\nabla}\cdot(\varepsilon_g \rho_g \vec{u_g}\vec{u_g}) = -\varepsilon_g \nabla \vec{P_g} + \underbrace{\vec{\nabla}\cdot\bar{\bar{\tau}}_g}_{\text{New term (Molecular Diffusion of Momentum)}} + \varepsilon_g \rho_g \vec{g} + \vec{F}_{dg} \qquad (4)$$

. where
- $P_g \equiv$ Gas Pressure
- $\bar{\bar{\tau}}_g \equiv$ shear stress tensor
- $\vec{g} \equiv$ gravitational acceleration
- $\vec{F}_{dg} \equiv$ Drag force for gas phase

- Moreover we have . $$\bar{\bar{\tau}}_g = \varepsilon_g \mu_g [\vec{\nabla}\vec{u_g} + (\vec{\nabla}\vec{u_g})^T] + \varepsilon_g [\lambda_g - \tfrac{2}{3}\mu_g]\vec{\nabla}\cdot\vec{u_g}\,\bar{\bar{I}} \qquad (5)$$

- $\mu_g \equiv$ shear gas viscosity
- $\lambda_g \equiv$ bulk " "
- $\bar{\bar{I}} \equiv$ unit tensor

$$\lambda_g = \tfrac{2}{3}\mu_g \qquad (6)$$

2) solid phase (fluid): $$\frac{\partial}{\partial t}(\varepsilon_s \rho_s \vec{u_s}) + \vec{\nabla}\cdot(\varepsilon_s \rho_s \vec{u_s}\vec{u_s}) = -\varepsilon_s \nabla \vec{P_g} + \underbrace{\vec{\nabla}\cdot\bar{\bar{\tau}}_s}_{\text{New}} - \underbrace{\vec{\nabla}P_s}_{\text{New}} + \varepsilon_s \rho_s \vec{g} + \vec{F}_{d,s} \qquad (7)$$

- where
  - P_s = solid pressure
  - $\vec{F}_{d,g}$ = Drag force for solid phase (8)
  - $\bar{\bar{\tau}}$ = shear stress tensor for solid phase

- $$\bar{\bar{\tau}}_s = \varepsilon_s \mu_s [\vec{\nabla}\vec{u_s} + (\vec{\nabla}\vec{u_s})^T] + \varepsilon_s [\lambda_s - \tfrac{2}{3}\mu_s]\vec{\nabla}\cdot\vec{u_s}\,\bar{\bar{I}} \qquad (9)$$

where
- → λ_s = bulk viscosity of solid phase
- → "Lun et al." : $$\lambda_s = \tfrac{4}{3}\varepsilon_s \rho_s d_s g_{o,ss}(1+e_{ss})\left(\frac{\theta_s}{\pi}\right)^{1/2} \qquad (10)$$
- → where
  - θ_s = granular temperature
  - e_{ss} = solid-solid restitution coefficient $\begin{cases} e_{ss} = 1 \\ e_{ss} = 0 \end{cases}$ (11)
  - $g_{o,ss}$ = radial distribution of solids: $g_{o,ss} = \left[1 - \left(\frac{\varepsilon_s}{\varepsilon_{s,max}}\right)^{1/3}\right]^{-1}$; $\varepsilon_{s,max} \cong .63$ (12)
  - $\varepsilon_{s,max}$ = maximum packing limit of solids
- → If
  - → $\varepsilon_s = \varepsilon_{s,max} \cong .63$ ⟹ Incompressible regime
  - → $\varepsilon_s < \varepsilon_{s,max}$ ⟹ compressible regime
  - → $\varepsilon_s \to 0$ ⟹ $g_{o,ss} \to 1$
  - → $\varepsilon_s \to \varepsilon_{s,max}$ ⟹ $g_{o,ss} \to \infty$

- where
  - $P_s \equiv$ solid Pressure $=$ (kinetic term) + (collisional Term) (14)
  - "Lun et. al." $\rightarrow$ For $\varepsilon_s < \varepsilon_{s,crit} \simeq .55$ $\quad P_s \triangleq \varepsilon_s \rho_s \theta_s + 2\rho_s(1+e_{ss})\varepsilon_s^2 g_{o,ss}\theta_s$ (15)
  - $\mu_s \triangleq \underbrace{\mu_{s,col}}_{\text{collisional}} + \underbrace{\mu_{s,kin}}_{\text{kinetic}} + \underbrace{\mu_{s,fric}}_{\text{friction}}$ (16)
  - where
    - $\mu_{s,col} = \frac{4}{5}\varepsilon_s \rho_s d_s g_{o,ss}(1+e_{ss})\left(\frac{\theta_s}{\pi}\right)^{1/2}$ (17)
    - $\mu_{s,kin} = \frac{10\rho_s d_s\sqrt{\pi\theta_s}}{96\,\varepsilon_s(1+e_{ss})g_{o,ss}}\left[1+\frac{4}{5}g_{o,ss}\varepsilon_s(1+e_{ss})\right]^2$ (18)
    - $\mu_{s,fric} = \frac{P_s \cdot \sin(\phi)}{2\sqrt{I_{2D}}}$ "Schaeffer relation" (19)
    - where
      - ϕ = angle of Internal friction (20)
      - I_{2D} = 2nd Invariant of deviatory stress tensor
      - $I_{2D} = \frac{1}{6}\left[(D_{11}-D_{22})^2+(D_{22}-D_{33})^2+(D_{33}-D_{11})^2\right]+D_{12}^2+D_{23}^2+D_{31}^2$ (21)
      - where
        - $D_{ij} = \frac{1}{2}\left[\frac{\partial u_{s,i}}{\partial x_j}+\frac{\partial u_{s,j}}{\partial x_i}\right]$ (22)
        - D_{ij} = rate of strain tensor
        - i,j = coordination
  - For $\varepsilon_s > \varepsilon_{s,crit} \cong .55$ $\Rightarrow$ Modification should be applied
  - now
    - $P_{s(new)} = P_{s(old)} + P_{fr}$ (24)
    - $\mu_{s(new)} = \mu_{s(old)} + \mu_{s,fr}$ (25)
    - "Johnson & Jackson":
      - $P_{fr} = Fr \times \frac{(\varepsilon_s-\varepsilon_{s,cr})^n}{(\varepsilon_{s,max}-\varepsilon_s)^p}$ (26)
      - where $Fr = .05$, $n = 2$, $p = 5$
      - $\mu_{fr} = P_{fr} \cdot \sin\phi$ (27)
      - where | $\varepsilon_{s,cr}$ = critical solid holdup
        | ϕ = angle of internal friction

- Drag Forces $(\vec{F}_{d,g}, \vec{F}_{s,g})$
  - By definition:
    - $\vec{F}_{d,g} = \beta_{sg}(\vec{u}_s - \vec{u}_g)$ (28)
    - $\vec{F}_{d,s} = \beta_{gs}(\vec{u}_g - \vec{u}_s)$ (29)
    - $\beta_{gs} = \beta_{sg} \equiv$ Momentum transfer coefficients between gas and solid (30)
  - Gidaspow eq.:

$$\beta_{sg} = \begin{cases} \frac{3}{4}C_D\frac{\varepsilon_s\varepsilon_g|\vec{u}_s-\vec{u}_g|\,\varepsilon_g^{-2.65}}{d_s} & \rightarrow \text{based on empirical correlation for } \varepsilon_g > .8 \\ 150\frac{\varepsilon_s(1-\varepsilon_g)\mu_g}{\varepsilon_g^2 d_s^2} + 1.75\frac{\rho_s\varepsilon_s|\vec{u}_s-\vec{u}_g|}{\varepsilon_s d_s} & \text{for } \varepsilon_g \leq .8 \rightarrow \text{based on Ergun's eq} \end{cases} \quad (31)$$

  - where
    - $C_D = \frac{24}{\varepsilon_g Re_s}\left[1+.15(\varepsilon_g Re_s)^{.887}\right]$ (32)
    - $Re_s = \frac{\rho_g d_s|\vec{u}_s-\vec{u}_g|}{\mu_s}$ (33)

Kinetic Theory of Granular Flow (KTGF)

- $\theta_s \equiv \frac{1}{3}\langle u_s'^2 \rangle$ "Granular Temeperature" (31)

where u_s' = fluctuating velocity of solids, $\langle\ \rangle \Rightarrow$ spatial average value

- Granular Temperature eq.: $\frac{3}{2}\left[\frac{\partial}{\partial t}(\varepsilon_s\rho_s\theta_s)+\vec{\nabla}.(\varepsilon_s\rho_s\vec{u}_s\theta_s)\right] = (-P_s\bar{\bar{I}}+\bar{\bar{\tau}}_s):\vec{\nabla}\vec{u}_s + \vec{\nabla}.(k_{\theta_s}\vec{\nabla}\theta_s) - \gamma_{\theta_s} + \phi_{gs}$ (35)

where
- k_{θ_s} = Diffusion coefficient of θ_s
- γ_{θ_s} = collisional dissipation energy
- ϕ_{gs} = Energy exchange between gas & solid phases
- $k_{\theta_s}\vec{\nabla}\theta_s$ = Diffusion flux of granular energy
- Gidaspow eq. for k_{θ_s}:

$$k_{\theta_s} = \frac{150\rho_s d_s\sqrt{\pi\theta_s}}{384(1+e_{ss})g_{o,ss}}\left[1+\frac{6}{5}\varepsilon_s g_{o,ss}(1+e_{ss})\right]^2 + 2\rho_s\varepsilon_s^2 d_s(1+e_{ss})g_{o,ss}\sqrt{\frac{\theta_s}{\pi}} \quad (36)$$

- "Lun et al." Eq. for γ_{θ_s}: $\gamma_{\theta_s} = \frac{12(1-e_{ss}^2)g_{o,ss}}{d_s\sqrt{\pi}}\rho_s\varepsilon_s^2\theta_s^{3/2}$ (37)
- for laminar flow: $\phi_{gs} = -3\beta_{gs}\theta_s$ (38) "Gidaspow et al. 1992"
- for disperse turbulent flow: $\phi_{gs} = \beta_{gs}\left[\sqrt{2k_f}\sqrt{3\theta_s} - 2k_f\right]$ (39) "Sinclair and Malla, 1998"

where k_f = Turbulent kinetic energy of the gas phase

Thermal energy eq. ⟶ for Nonisothermal F.B.s.

I) Gas phase: $\frac{\partial}{\partial t}(\varepsilon_s\rho_g\Delta\hat{H}_g)+\vec{\nabla}.(\varepsilon_g\rho_g\vec{u}_g\Delta\hat{H}_g) = -\varepsilon_g\frac{\partial P_g}{\partial t}+\bar{\bar{\tau}}_g:\vec{\nabla}\vec{u}_g - \vec{\nabla}.\vec{q}_g + \phi_{sg} + \Delta H_{rxn}$ (40)

where: $\Delta\hat{H}_g$ = specific enthalpy of gas phase

II) Solid phase: $\frac{\partial}{\partial t}(\varepsilon_s\rho_s\Delta\hat{H}_s)+\vec{\nabla}.(\varepsilon_s\rho_s\vec{u}_s\Delta\hat{H}_s) = -\varepsilon_s\frac{\partial P_s}{\partial t}+\bar{\bar{\tau}}_s:\vec{\nabla}\vec{u}_s - \vec{\nabla}.\vec{q}_s + \phi_{gs}$ (41)

where
- q_g = flux of diffusional thermal energy for gas phase
- q_s = 〃 〃 〃 〃 〃 〃 solid 〃
- ϕ_{sg} & ϕ_{gs} = heat transfer between gas & solid phases ($\phi_{sg} = -\phi_{gs}$) (42)
- ΔH_{rxn} = Heat of reaction

$$\Delta\hat{H}_g = \int_{T_{g,ref}}^{T_g} c_{pg}\,dT_g + \Delta\hat{H}_{f,g} \quad (43) \quad , \quad \Delta\hat{H}_s = \int_{T_{s,ref}}^{T_s} C_{ps}\,dT_s + \Delta\hat{H}_{f,s} \quad (44)$$

where

$$\phi_{sg} = h_{sg}\,\Delta T = h_{sg}(T_s - T_g) \quad (45)$$

where
- h_{sg} = heat transfer coefficient between gas & solid phase
- $h_{sg} = \frac{6k_g\varepsilon_s\varepsilon_g Nu_s}{d_s^2}$ (46)

- where
  - k_g = thermal conductivity of gas phase
  - Nu_s = Nusselt No. for solid phase
  - typical example for Nu_s "Gunn": (for $.35 \leq \varepsilon_g \leq 1.0$)

$$Nu_s = (7 - 10\varepsilon_g + 5\varepsilon_g^2)(1 + .7 Re_s^{.2} Pr^{1/3}) + (1.33 - 2.4\varepsilon_g + 1.2\varepsilon_g^2) Re_s^{.7} Pr^{1/3} \quad (47)$$

where: $Pr = \dfrac{c_{Pg}\,\mu_g}{k_g}$ (48)

Species Continuity Equation

- for species "i" in the gas phase, Mass Transfer Operation:

$$\frac{\partial C_i}{\partial t} + \vec{\nabla}\cdot\vec{N_i} = R_i \quad (49)$$

R_i: Net rate of gen/con. of "i" per unit volume

$\vec{N_i}$: Total flux of "i": $\vec{N_i} = C_i\vec{u} + \vec{J_i} = C_i\vec{u_i}$

- in a FB, we have:

$$\frac{\partial}{\partial t}(\underbrace{\varepsilon_g \rho_g Y_{i,g}}) + \vec{\nabla}\cdot(\varepsilon_g \rho_g Y_{i,g}\vec{u_g}) = -\vec{\nabla}\cdot(\varepsilon_g \vec{J_{gi}}) + \varepsilon_g R_{gi} \quad (50)$$

$= \rho_{gi}$ "mass concentration of "i" in the gas phase"

- in addition:
  - $\sum_{i=1}^{N_s} Y_{gi} = 1$ (51), $N_s \equiv$ Total No. of "i" components
  - $\vec{J_{gi}} = -\left[\rho_g D_{g,im} + \dfrac{\mu_t''}{Sc_t}\right]\vec{\nabla} Y_{gi} - D_{T,i}\dfrac{\vec{\nabla} T_g}{T_g}$ (52)

$D_{g,im} \equiv$ Molecular diffusion of "i" in mixture

$D_{T,i} \equiv$ " " " , due to $\vec{\nabla} T_g$

$\mu_t \equiv$ Turbulent viscosity (Model)

$Sc_t \equiv$ " Schmidt No. $= \dfrac{\mu_t}{\rho D_t}$ (Generally we get: $Sc_t \cong .7$)

- Turbulence eqs.:

$$\bar{u_i} = \frac{1}{\Delta t}\int_0^{\Delta t} u_i\,dt \qquad \begin{cases} u_i = \bar{u}_i + u_i' \\ P = \bar{P} + P_i' \\ T = \bar{T} + T_i' \end{cases}$$

where $(i = x, y, z)$ N-S eqs. (53)

fluctuating variables

- k-ε model [by Launder & Spalding], a semi-empirical
- Modified version of "Standard k-ε model", called "RNG k-ε" for multiphase flow
- k eq. "Turbulent kinetic energy"

$$\frac{\partial}{\partial t}(\rho_m k) + \vec{\nabla}\cdot(\rho_m \vec{u_m} k) = \vec{\nabla}\cdot(\alpha_k \mu_t \vec{\nabla} k) + G_{k,m} - \rho_m \varepsilon \quad (54)$$

- ε eq. "Dissipation rate"

$$\frac{\partial}{\partial t}(\rho_m \varepsilon) + \vec{\nabla}\cdot(\rho_m \vec{u_m}\varepsilon) = \vec{\nabla}\cdot(\alpha_\varepsilon \mu_t \vec{\nabla}\varepsilon) + \frac{\varepsilon}{k}(C_{1s} G_{k,m} - C_{2s}\rho_m \varepsilon) - R_s \quad (55)$$

where
- α_k, α_t = Inverse effective Prandtl No. for k & ε (56)
  for Re↑: $\alpha_k = \alpha_\varepsilon \cong 1.393$
- $G_{k,m}$ = generation of turbulent kinetic energy due to mean velocity gradient
  $= G_k + \underbrace{G_b}_{small} \Rightarrow G_{k,m} = -\rho u_i' u_j' \dfrac{\partial u_j}{\partial x_i}$ (57)
- $C_{1s} = 1.42$, $C_{2s} = 1.68$, R_s = Rate of dissipation (**)
- for high Re No.
  - $\mu_t = \rho_m C_\mu \dfrac{k^2}{\varepsilon}$ (58)
  - $C_\mu = .0845$ whereas in standard k-ε model: $C_\mu = .09$

- for α_k & α_ε : $$\left|\frac{\alpha - 1.3929}{\alpha_0 - 1.3929}\right|^{0.6321} \times \left|\frac{\alpha + 2.3929}{\alpha_0 + 2.3929}\right| = \frac{\mu_{mol}}{\mu_{eff}} \quad (59)$$

where $\alpha_0 = 1$

. for high Re No. $\left(\frac{\mu_{mol}}{\mu_{eff}} << 1\right)$, $\alpha_k = \alpha_\varepsilon = 1.393$ (60)

. If $\hat{v} = \frac{\mu_{eff}}{\mu}$ (61) $\Rightarrow$ $$\frac{d}{d\hat{v}}\left(\frac{\rho^2 k}{\sqrt{\mu \varepsilon}}\right) = 1.72 \frac{\hat{v}}{\sqrt{\hat{v}^3 - 1 + C_v}} \quad (62) \Rightarrow \begin{cases} \mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \\ C_\mu = .0845 \\ C_v \cong 100 \end{cases} \quad (63)$$

→ In high Re No. : $$R_s = \frac{C_\mu \rho \eta^3 (1 - \eta/\eta_0) \varepsilon^2}{(1 + \beta \eta^3) k} \quad (64)$$

$$\begin{cases} \eta = S \frac{k}{\varepsilon} \\ \eta_0 = 4.38 \\ \beta = .012 \\ S \equiv \text{modulus of mean rate of strain tensor} = \sqrt{2 S_{ij} S_{ij}} \\ S_{ij} = \frac{1}{2}\left\{\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right\} \end{cases} \quad (65)$$

▨ ICs & BCs :

- at the outlet of FB: $P = 1^{atm}$
- at the wall for gas-phase: No-slip condition
- for the solid phase at wall : (slip)
  - In 80s, Johnson & Jackson : $$\tau_s = \frac{-\pi}{6}\sqrt{3\phi}\,\frac{\varepsilon_s}{\varepsilon_{s,max}} \rho_s g_0 \sqrt{\theta_s}\, \vec{u}_{s,para} \quad (66)$$

where
→ $u_{s,para}$ ≡ particle slip velocity parallel to the wall
→ ϕ ≡ specularity coefficient between the particle & the wall

ϕ [0 ≡ smooth walls — 1 ≡ rough walls]

for $\phi \to 0$: a free slip BC.

- JJ (1987) for the total Granular heat flux:

$$q_s = \frac{\pi}{6}\sqrt{3\phi}\,\frac{\varepsilon_s}{\varepsilon_{s,max}} \rho_s g_0 \sqrt{\theta_s}\, \vec{u}_{s,para} - \frac{\pi}{4}\sqrt{3}\,\frac{\varepsilon_s}{\varepsilon_{s,max}}(1 - e_{sw}^2)\rho_s g_0 \theta_s^{3/2} \quad (67)$$

e_{sw} ≡ particle-wall restitution coefficient that means the dissipation of solids turbulent kinetic energy by collision with the wall ϕ & e_{sw}